Basic Circuit Analysis

W. Bolton, "Mechatronics --- Electronic control systems in mechanical and electrical engineering," 5th edition, Pearson Education Limited 2012

J. Edward Carryer, R. Matthew Ohline, Thomas W. Kenny, "Introduction to Mechatronic Design," Prentice Hall 2011, Chap 9

線上學習網站：https://www.electronics-tutorials.ws

PowerPoint 中部分圖片擷取和修改自教科書和網路圖片

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Terminology -1

- **Charge (Q)**: quantity of charged particles
  - Unit: Coulomb (C), $1C = 6.24 \times 10^{18}$ charged particles
- **Current (I)**: number of charged particles that move past any point in an increment of time --- $\frac{dQ}{dt}$
  - Unit: Ampere ($A = \frac{C}{s}$)
- **Voltage (V)**: strength of the electric field --- $\frac{dW}{dQ}$, work done per unit charge
  - Unit: volt ($V = \frac{J}{C}$, work done in moving a coulomb of charge)

Terminology -2

- Power (P): rate at which work is being done

$$P = \frac{work}{time} = \frac{joules\ (J)}{second(s)} = \frac{joules}{coulomb} \times \frac{coulomb}{second} = \frac{J}{C} \times \frac{C}{s} = VI$$

  - Unit: watt ($W = \frac{J}{s}$)
  - $1W = 1\frac{J}{s} = 1\frac{Nm}{s} = 1\frac{(kg\frac{m}{s^2})m}{s} = 1\frac{kgm^2}{s^3}$ (ML^2T^{-3})

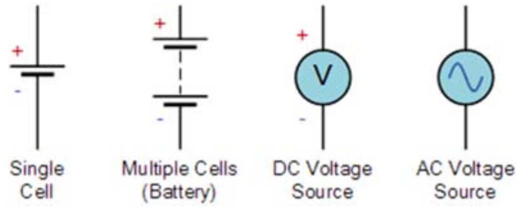
represented in seven SI base units

Seven SI Base Units

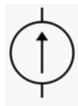
Name	Symbol	Measure	Dimension Symbol
Meter	m	Length	L
Kilogram	kg	Mass	M
Second	s	Time	T
Ampere	A	Electric current	I
Kelvin	K	Thermodynamic temperature	Θ
Mole	mol	Amount of substance	N
Candela	cd	Luminous intensity	J

Terminology -3

- **Voltage source**: an ideal source would deliver a constant voltage independent of current drawn from the source
  - Direct current (DC)
  - Alternating current (AC)



- **Current source**: an ideal current source would generate whatever voltage was necessary to produce a constant flow of current



Terminology -4

- Circuits
  - A complete circuit

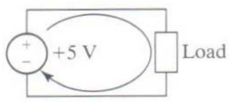


FIGURE 9.2
The simplest complete circuit.

  - An open circuit

FIGURE 9.3
In an open circuit, no current flows.

  - A short circuit

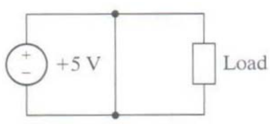


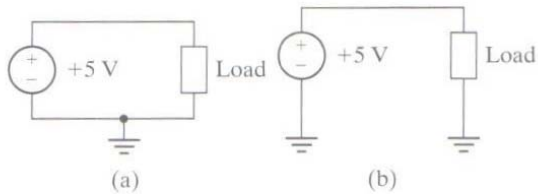
FIGURE 9.4
A short circuit across the voltage source and the load.

Terminology -5

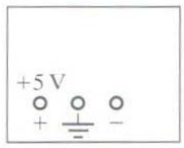
- **Shorting**: the process of creating a short circuit

  a transient effect, causing high current to flow

- **Ground**: a common reference point through a circuit and represents the point that will be assigned a value of 0V

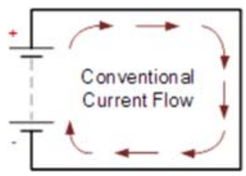


- **Floating**: no direct connection to the building ground

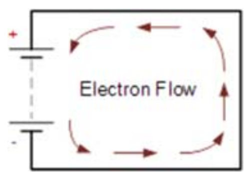


Electrical Current

- Conventional current flow
  - The flow of positive charge around a circuit, being positive to negative



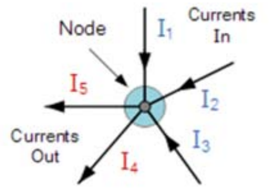
- Electron flow
  - The actual current flowing in an electrical circuit is composed of electrons that flow from the to positive



Kirchhoff's Law

- Kirchhoff's Current (First) Law
  - The sum of all the current entering a node of a circuit is the same as the sum of all the current leaving that same node
  - Conservation principle

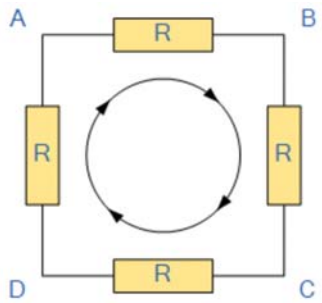
$$\Rightarrow \sum_{k=1}^{n} i_k = 0$$



$I_1 + I_2 + I_3 + (-I_4 + -I_5) = 0$

- Kirchhoff's Voltage (Second) Law
  - The sum of the voltage differences around any closed loop in a circuit is zero

$$\Rightarrow \sum_{k=1}^{n} V_k = 0$$

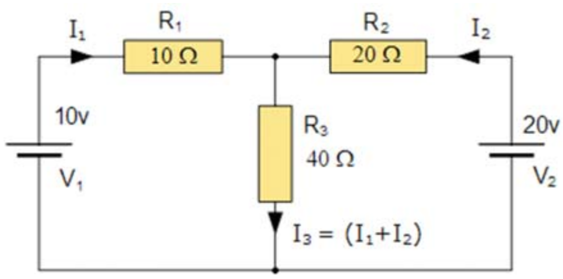


$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$

An Example -1

- Ex: A circuit
  - Method 1: Kirchhoff's Laws

$$10 - 10i_1 - 40i_3 = 0$$
$$20 - 20i_2 - 40i_3 = 0$$
$$i_3 = i_1 + i_2$$

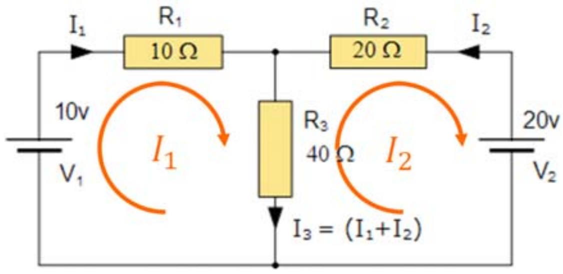


$$\Rightarrow i_1 = -0.143 \quad i_2 = 0.429 \quad i_3 = 0.286$$

20V battery is charging the 10V battery

An Example -2

- Ex: A circuit
  - Method 2: Mesh current analysis (or loop analysis or Maxwell's circulating currents)
    - Label inside loops in a clockwise direction with circulating currents as the aim to cover all the elements of the circuit at least once



$$I_1 = i_1, I_2 = -i_2$$

$$10 = (10 + 40)I_1 - 40I_2$$

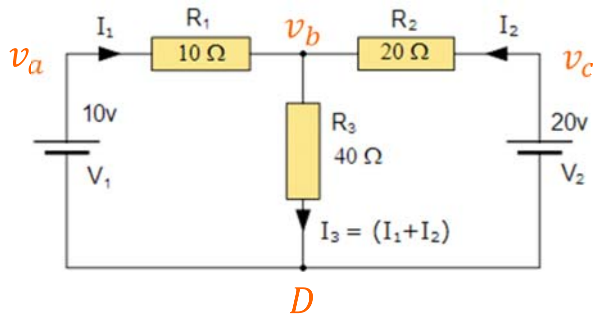
$$-20 = -40I_1 + (20 + 40)I_2$$

$$V = RI$$

$$\Rightarrow \quad I = R^{-1}V = \begin{bmatrix} -0.143 \\ -0.429 \end{bmatrix}$$

An Example -3

- Ex: A circuit
  - Method 3: Nodal voltage analysis



$$i_1 + i_2 = i_3$$

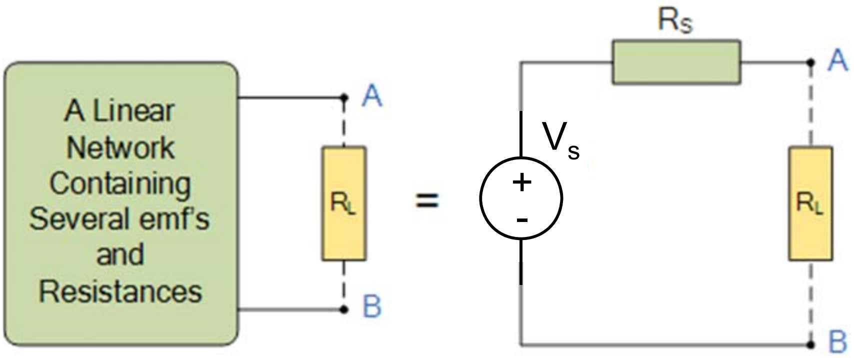
$$\frac{v_a - v_b}{10} + \frac{v_c - v_b}{20} = \frac{v_b}{40}$$

$$v_a = 10, v_c = 20$$

$$\Rightarrow \quad v_b = \frac{80}{7}V \qquad i_3 = \frac{2}{7} = 0.286A$$

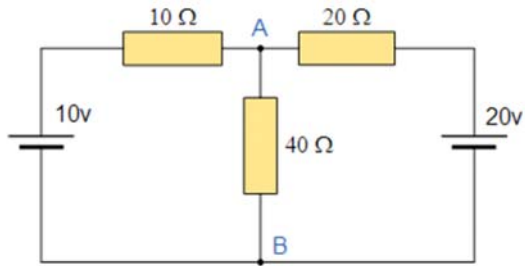
Thevenin's Theorem

- Thevenin's Theorem
  - Any linear circuit containing several voltages and resistors can be replaced by just one single voltage in series with a single resistance connected across the load

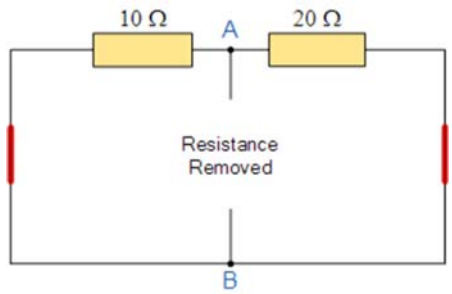


Revisit the Example -1

- Ex: The circuit



  - Step 1: Remove the load resistor

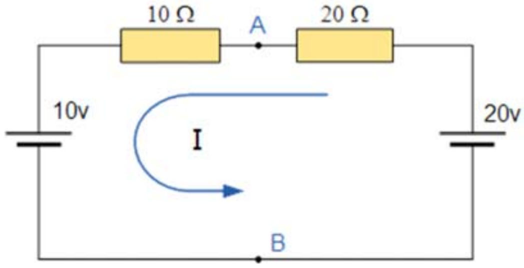


  - Step 2: Find the equivalent resistor by shorting all voltage sources and opening circuiting all current sources

$$R_{equiv} = \frac{1}{\frac{1}{10}+\frac{1}{20}} = 6.67\Omega$$

Revisit the Example -2

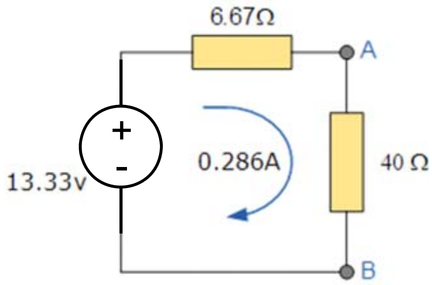
- Step 3: Find the equivalent voltage source by the usual circuit analysis



$$I = \frac{V}{R} = \frac{20 - 10}{20 + 10} = 0.33A$$

$$V_{AB} = 20 - (20 * 0.33) = 13.33V$$
$$= 10 + (10 * 0.33) = 13.33V$$

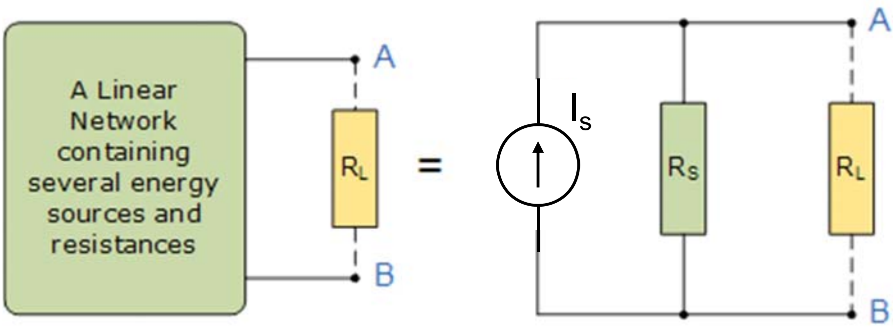
- Step 4: Find the current flowing through the load resistor



$$I = \frac{V}{R} = \frac{13.33}{6.67 + 40} = 0.286A$$

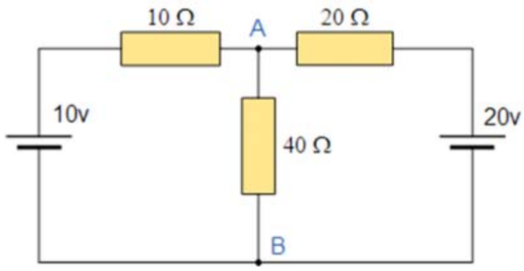
Norton's Theorem

- Norton's Theorem
  - Any linear circuit containing several energy sources and resistances can be replaced by a single constant current generator in parallel with a single resistor

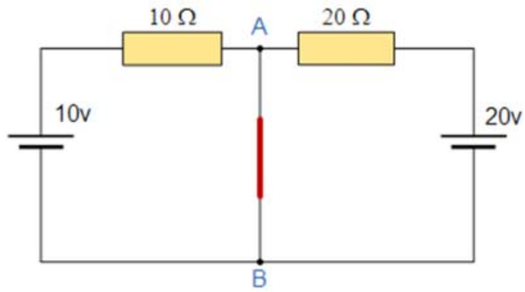


Revisit the Example -3

- Ex: The circuit



  - Step 1: Remove the load resistor

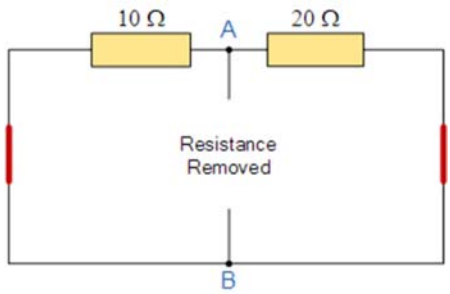


  - Step 2: Find the equivalent current source by placing a short link on the original load terminals

$$I_{equiv} = \frac{10}{10} + \frac{20}{20} = 2A$$

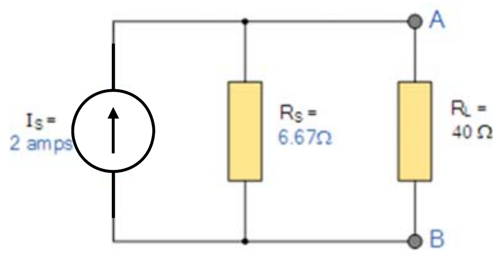
Revisit the Example -4

- Ex: Revisit the same circuit
  - Step 3: Find the equivalent resistor by shorting all voltage sources and opening circuiting all current sources



$$R_{equiv} = \frac{1}{\frac{1}{10} + \frac{1}{20}} = 6.67\Omega$$

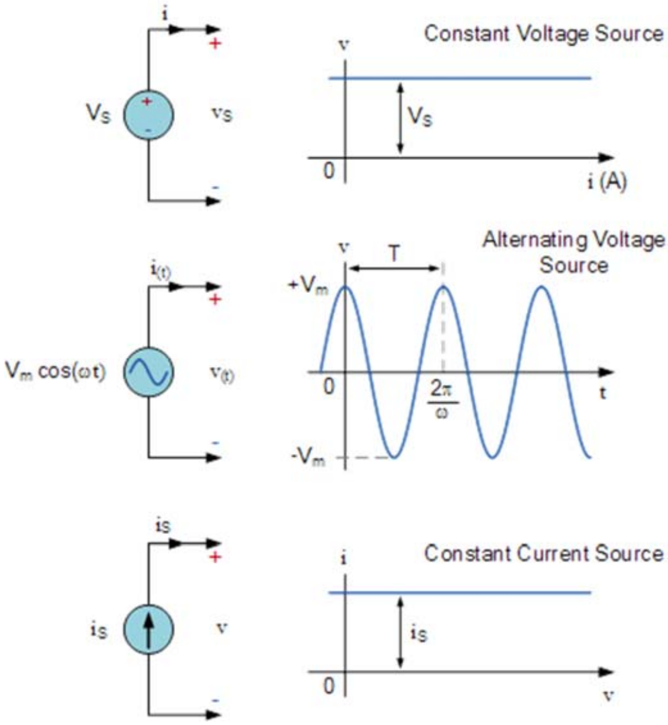
  - Step 4: Find the current flowing through the load resistor



$$I = \frac{6.67}{6.67 + 40} * 2 = 0.286A$$

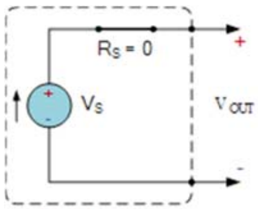
Electrical Sources

- Electrical Sources
  - DC voltage source
  - AC voltage source
  - Current source

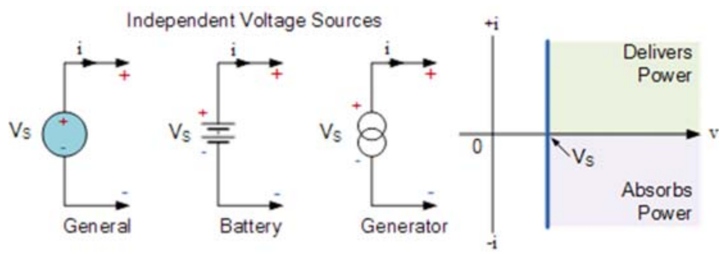


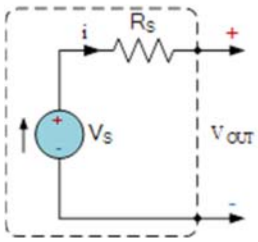
Voltage Sources

- Ideal and practical voltage source

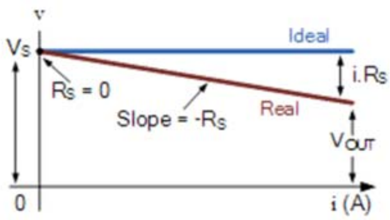


Ideal voltage source





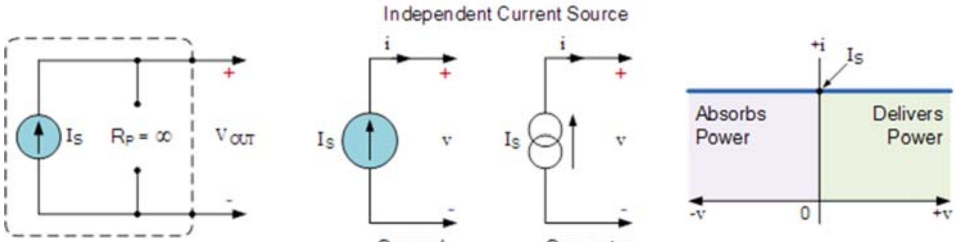
Practical voltage source



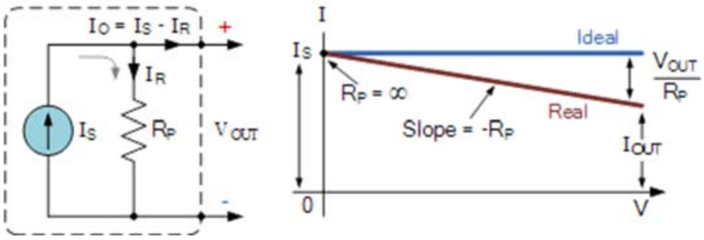
$$V_{out}(I) = V_s - IR_s$$

Current Sources

- Ideal and practical current source



Ideal voltage source



Practical current source

$$I_{out}(V_{out}) = I_s - \frac{V_{out}}{R_p}$$

The End

- Questions?