Locomotion

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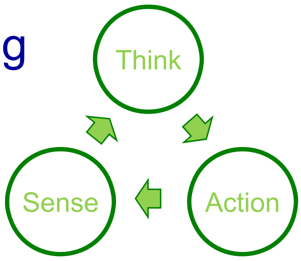
R. Siegwart and I. R. Nourbakhsh, Thomas W. Kenny, “Introduction to Autonomous Mobile Robots,” The MIT Press 2004
PowerPoint 中部分圖片擷取和修改自教科書和網路圖片

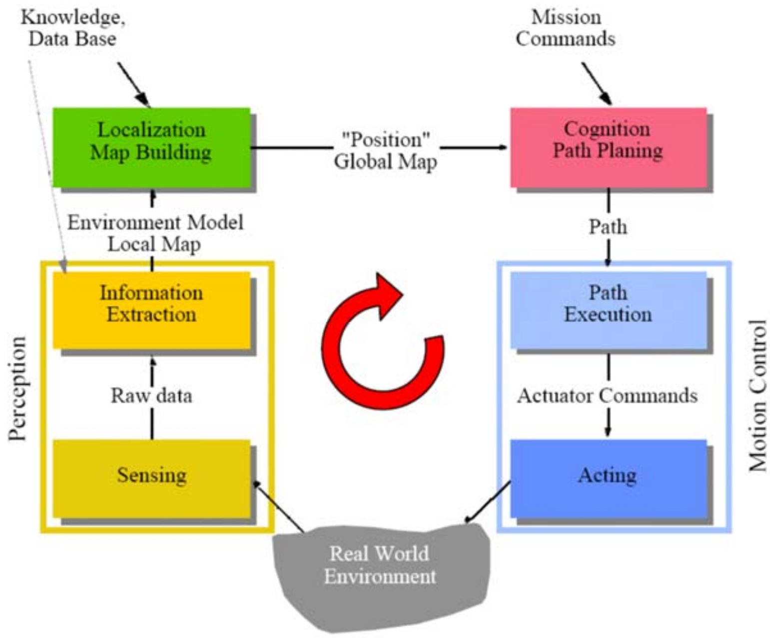
Key Questions

- The three key questions in Mobile Robotics
  - Where am I?
  - Where am I going?
  - How do I get there?
- To answer these questions, the robot has to
  - Be able to “move”
  - Have a model of the environment (given or autonomously built)
  - Perceive and analyze the environment
  - Find its position within the environment
  - Plan and execute the movement

Important Subtasks for Great Mobility

- Mechanisms that enable locomotion
- Perception and cognition
  - Sensing + sensory data interpretation
- Localization and Mapping
- Navigation and Planning



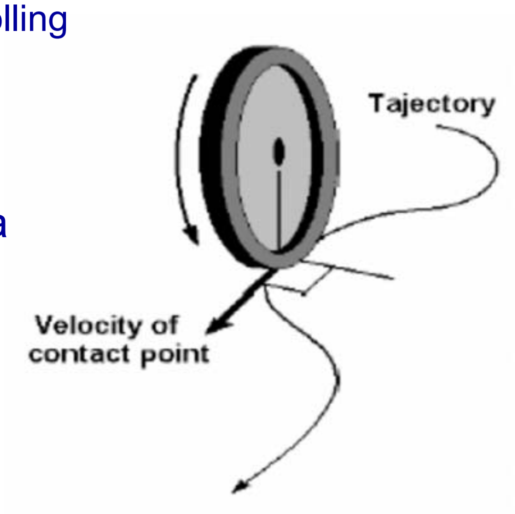


Mobile Robots with Wheels

- Wheels are the straight forward solutions for most applications
- Three wheels are sufficient to guarantee stability
  - With more than three wheels, a flexible suspension is required
- Selection of wheels depends on the application

Idealized Rolling Wheels

- Assumptions
  - Rigid wheel
  - No side slip
  - In the direction orthogonal to that of rolling
  - No translational slip
  - between the wheel and the floor
- For low velocity, idealized rolling is a reasonable wheel model
- Parameters
  - r: radius
  - v: linear velocity
  - w: angular velocity
  - t: steering velocity



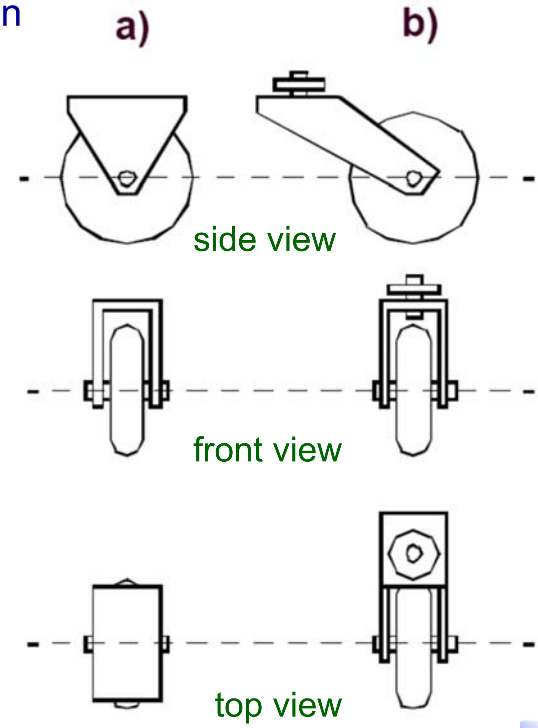
Non-slipping and pure rolling

P.S. An interesting Research

- Gyrover
  - A single-wheel, gyroscopically stabilized robot

Four Basic Wheels Types -1

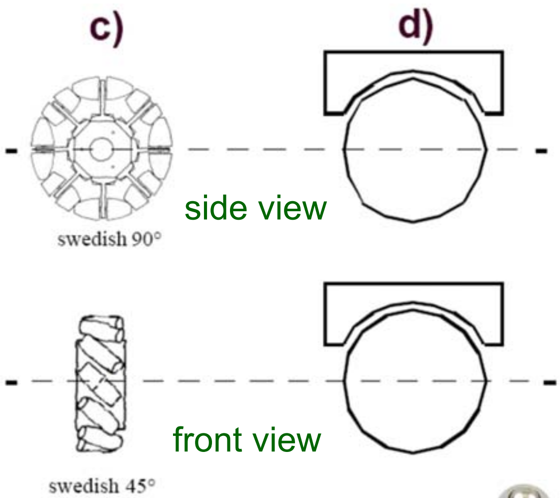
- a) Fixed/steered standard wheel
  - One/two degrees of freedom; rotation around the (motorized) wheel axle (and the contact point)



- b) Castor wheel
  - Three degrees of freedom; rotation around the wheel axle, the contact point and the castor axle

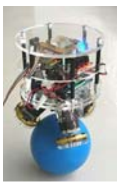
Four Basic Wheels Types -2

- c) Swedish wheel
  - Three degrees of freedom; rotation around the (motorized) wheel axle, around the rollers and around the contact point

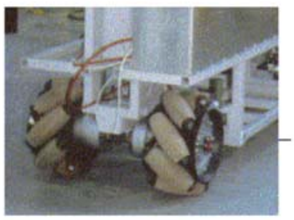


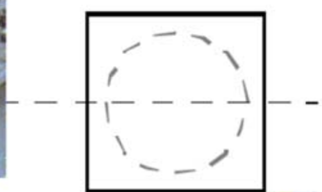
- d) Spherical wheel

CMU Ball bot



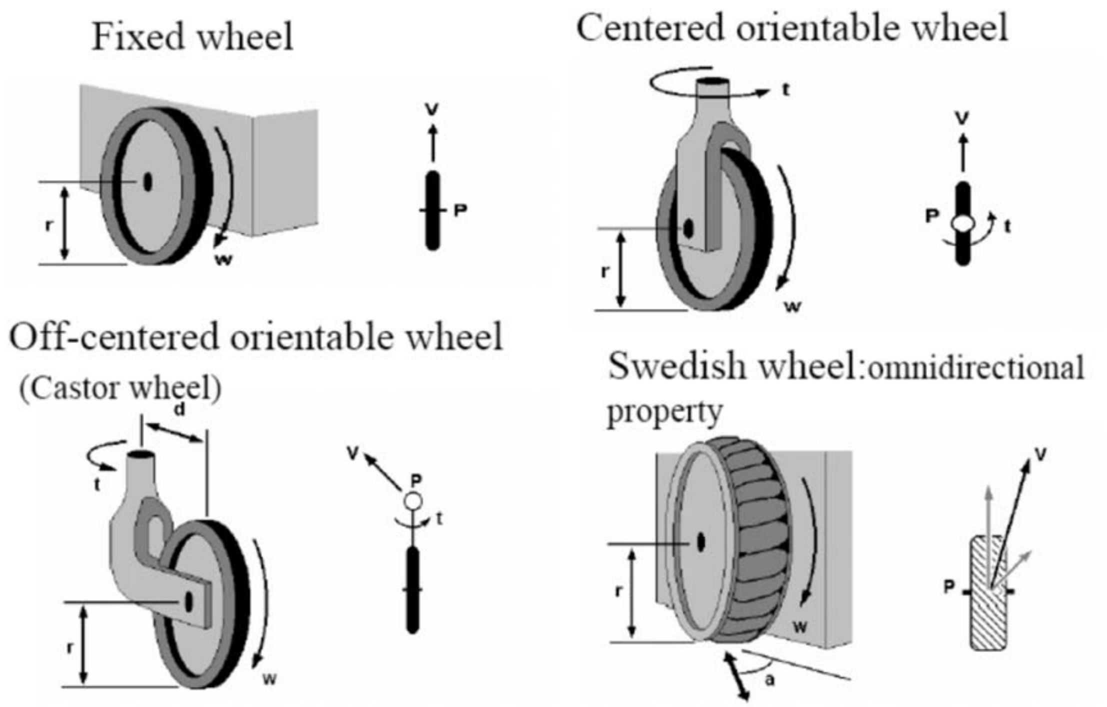
BallIP





Four Basic Wheels Types -3

- With parameters



Four Basic Wheels Types -4

- Fixed standard wheel

  – Velocity of point **P**

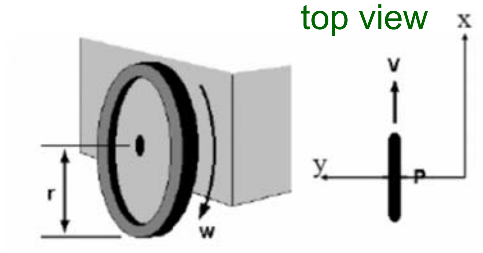
$$V = (r \times w)a_x$$

where, a_x : A unit vector to X axis

  – Restriction to the robot mobility

Point **P** cannot move to the direction perpendicular to plane of the wheel.

top view



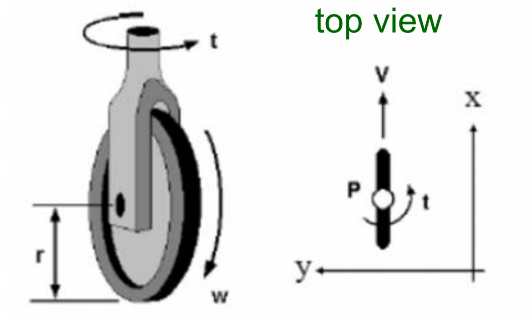
Four Basic Wheels Types -5

- Steered standard wheel
  - Velocity of point **P**

$$V = (r \times w)a_x$$

where, a_x : A unit vector of x axis
a_y : A unit vector of y axis

  - Restriction to the robot mobility



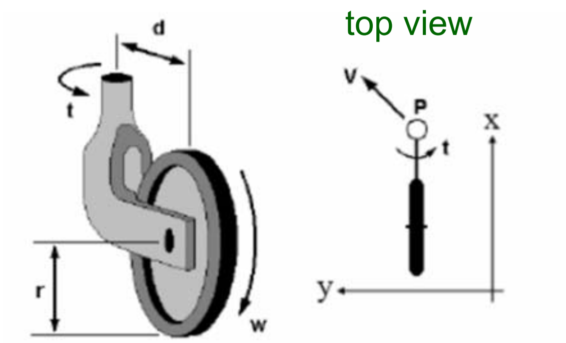
Four Basic Wheels Types -6

- Caster wheel
  - Velocity of point **P**

$$v = (r \times w)a_x + (d \times t)a_y$$

where, a_x : A unit vector of x axis
a_y : A unit vector of y axis

  - Restriction to the robot mobility



Four Basic Wheels Types -7

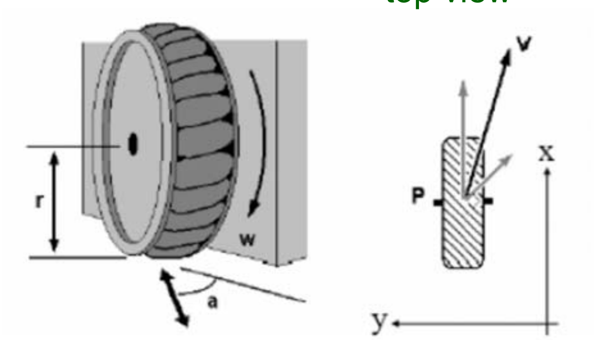
- Swedish wheel
  - Velocity of point **P**

$$v = (r \times w)a_x + Ua_s$$

where, a_x : A unit vector of x axis
a_s : A unit vector to the motion of roller

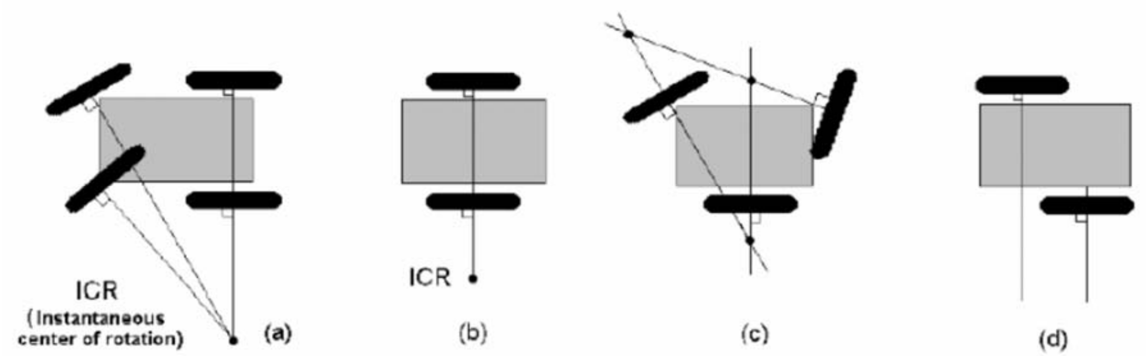
  - Omnidirectional property

top view



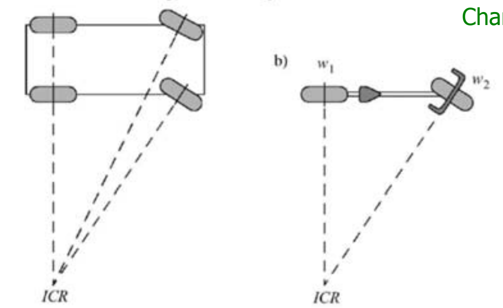
Locomotion Characteristics -1

- Instantaneous center of rotation (ICR)
  - A cross point of all axes of the wheels



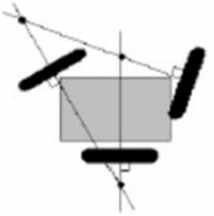
Differential drive, Changeable ICR

ICR at infinity, Moving straight



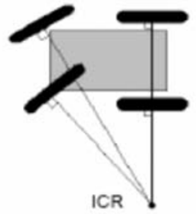
Locomotion Characteristics -2

- Degree of mobility, δ_m
  - The DOF of the robot motion



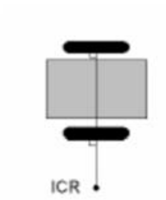
Cannot move anywhere (No ICR)

- Degree of mobility : 0



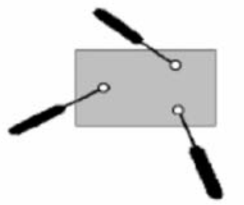
Fixed arc motion (Only one ICR)

- Degree of mobility : 1



Variable arc motion (line of ICRs)

- Degree of mobility : 2



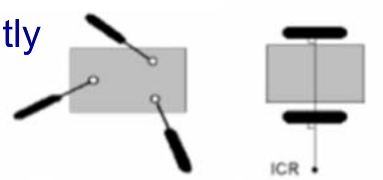
Fully free motion

(ICR can be located at any position)

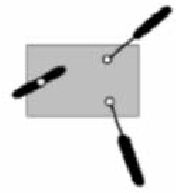
- Degree of mobility : 3

Locomotion Characteristics -3

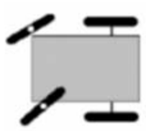
- Degree of steerability, δ_s
  - The number of steered standard wheel that can be steered independently



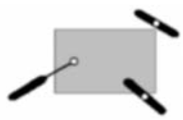
No centered orientable wheels
Degree of Steerability: 0



One centered orientable wheel
Degree of Steerability: 1



Two mutually dependent centered orientable wheels
Degree of Steerability: 1

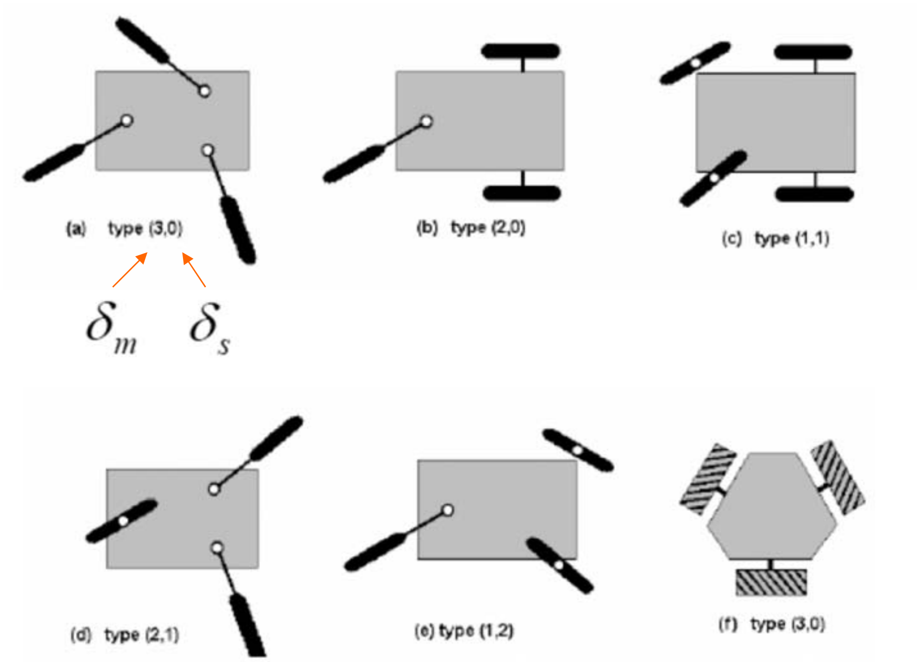


Two mutually independent centered orientable wheels
Degree of Steerability: 2

Locomotion Characteristics -4

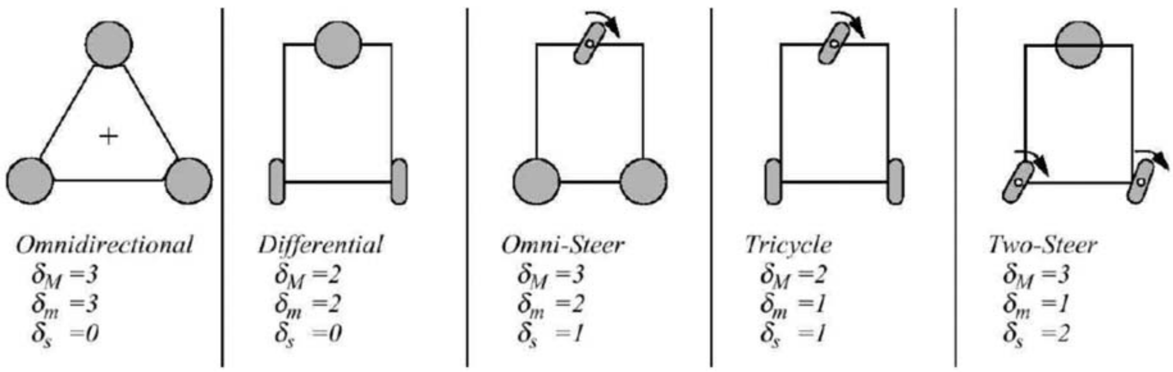
- Degree of maneuverability, $\delta_M = \delta_m + \delta_s$

Examples of robot types (degree of mobility, degree of steerability)



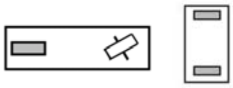
Locomotion Characteristics -5

- Five basic types of three-wheel configuration



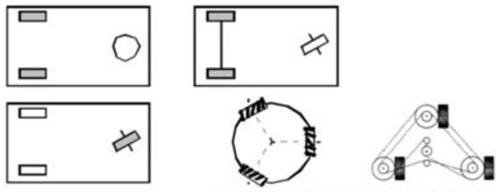
Locomotion Characteristics -6

- Different arrangements of wheels
  - Two wheels

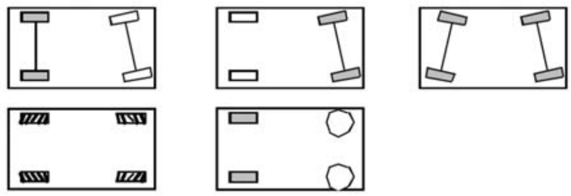


Grey: motorized steering

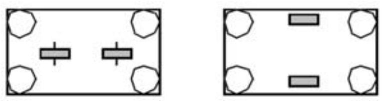
  - Three wheels



  - Four wheels



  - Six wheels



Locomotion Characteristics -7

- Comments
  - Stability of a vehicle is guaranteed with 3 wheels
    - Center of gravity is located within the triangle formed by ground contact points of the wheels
    - Stability is improved by 4 and more wheels
      - However, this arrangements are hyper-static and require a flexible suspension system
  - Bigger wheels allow to overcome higher obstacles
    - Higher torque at wheel shaft is required
  - Most arrangements are non-holonomic
    - Require high control effort
  - Combining actuation and steering on one wheel makes the design complex and adds additional errors for odometry

Locomotion Characteristics -8

- Holonomic constraints (configuration constraints)

$$\varphi_j(q_1, q_2, \cdots, q_n, t) = 0\,, j = 1,2, \cdots, m$$

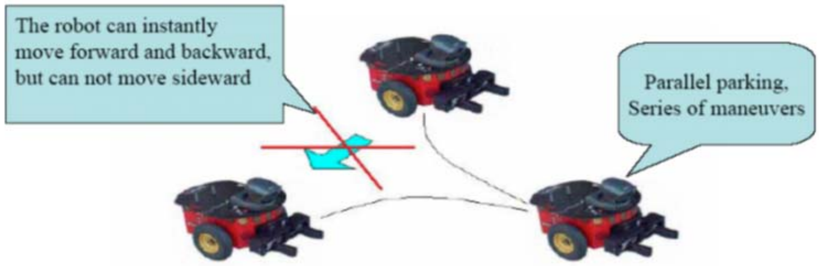
q_i: generalized coordinate

- Non-holonomic constraints (velocity constraints)

NOT integrable

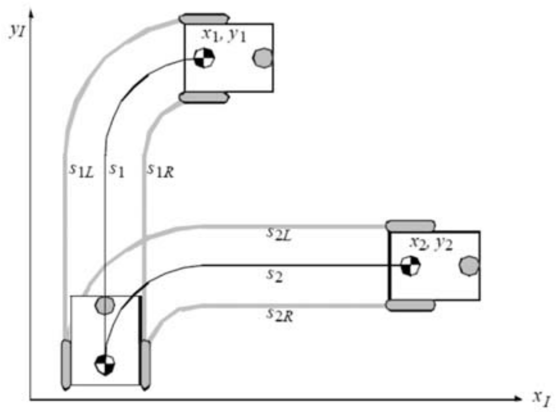
$$\sum_{i=1}^{n} a_{ji}(q_1, q_2, \cdots, q_n, t)\, \dot{q}_i + b_j(q_1, q_2, \cdots, q_n, t) = 0$$

$$\sum_{i=1}^{n} a_{ji}(q_1, q_2, \cdots, q_n, t) dq_i + b_j(q_1, q_2, \cdots, q_n, t) dt = 0$$ Pfaffian form



Locomotion Characteristics -9

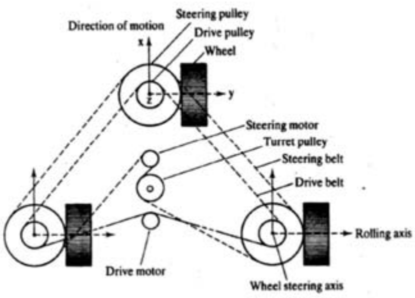
- Non-Holonomic Systems
  - Differential equations are not integrable to the final position
  - The measure of the traveled distance of each wheel is not sufficient to calculate the final position of the robot
    - One has also to know how this movement was executed as a function of time



Some Robots -1

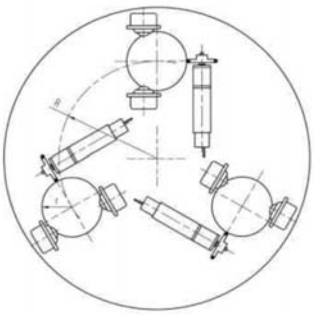
- Synchro drive
  - All wheels actuated synchronously by one motor
    - Defines the speed of the vehicle
  - All wheels steered synchronously by a second motor
    - Sets the heading of the vehicle
  - Fixed orientation of the robot frame in world frame

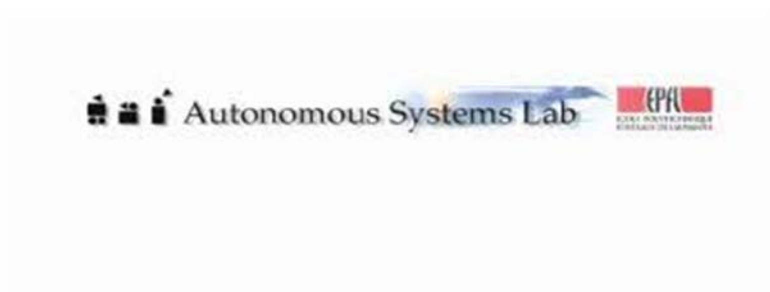
$$\delta_M = \delta_m + \delta_S = 1 + 1 = 2$$

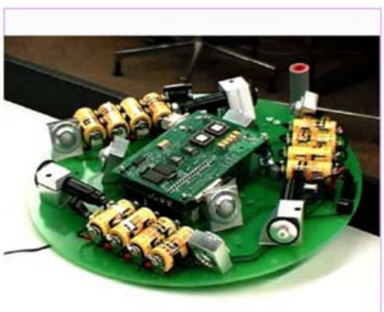


Some Robots -2

- Tribolo
  - An omnidirectional robot with 3 spherical wheels

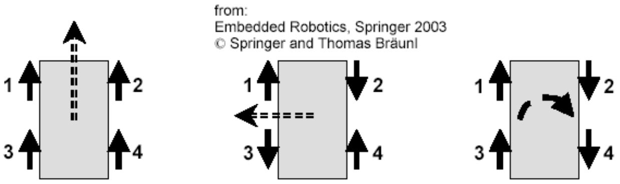


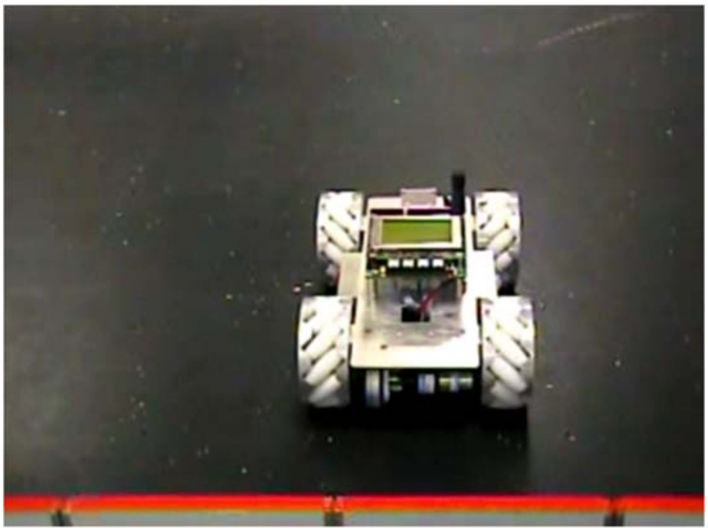




Some Robots -3

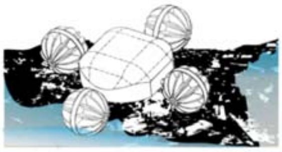
- A robot
  - An omnidirectional robot with 4 Swedish wheels

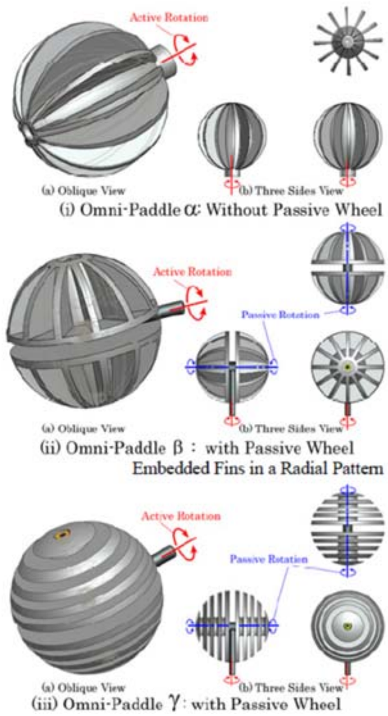




Some Robots -4

- Omnidirectional paddle





Mobile Robot Kinematics -1

- Aim
  - Description of mechanical behavior of the robot for design and control
    - Similar to manipulator kinematics
  - However, mobile robot can move unbounded with respect to its environment
    - There is no direct way to measure the robot's position
    - Position must be integrated over time
    - Leads to inaccuracies of the position (motion) estimates
      - The number 1 challenge in mobile robotics
  - Understanding mobile robot motion starts with understanding wheel constraints placed on the robots mobility

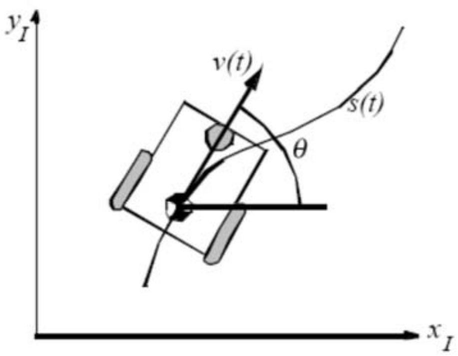
Mobile Robot Kinematics -2

- Kinematic model
  - Establish the robot speed $\dot{\xi} = \begin{bmatrix}\dot{x} & \dot{y} & \dot{\theta}\end{bmatrix}^T$ as a function of the wheel speeds $\dot{\varphi}_i$, steering angles β_i, steering speeds $\dot{\beta}_i$ and the geometric parameters of the robot (configuration coordinates)
  - Forward kinematics

$$\dot{\xi} = \begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = f(\dot{\varphi}_1, \ldots \dot{\varphi}_n, \beta_1, \ldots \beta_m, \dot{\beta}_1, \ldots \dot{\beta}_m)$$

  - Inverse kinematics

$$\begin{bmatrix}\dot{\varphi}_1 & \cdots & \dot{\varphi}_n & \beta_1 & \ldots & \beta_m & \dot{\beta}_1 & \ldots & \dot{\beta}_m\end{bmatrix}^T = f(\dot{x}, \dot{y}, \dot{\theta})$$



Mobile Robot Kinematics -3

- FYI: Manipulator Kinematics
  - Forward kinematics of manipulators

    Given θ_i in ${}^{w}_{i}T$, calculate ${}^{w}\{H\}$ or ${}^{w}P$

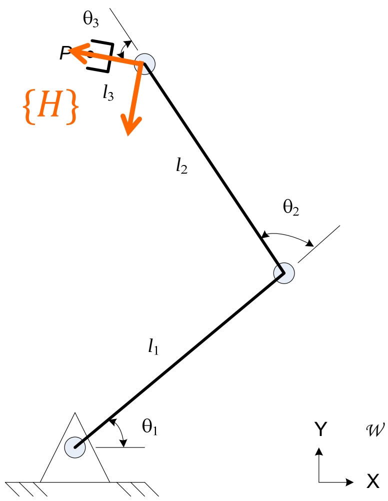
    $${}^{w}_{H}T = f(\theta_1, \ldots, \theta_i, \ldots, \theta_n)$$

    $${}^{w}P = {}^{w}_{i}T\ {}^{i}P\ , i = 1, \ldots, n$$

  - Inverse kinematics – This chapter

    Given ${}^{w}\{H\}$ or ${}^{w}P$, calculate θ_i in ${}^{w}_{i}T$

    $$[\theta_1, \ldots, \theta_i, \ldots, \theta_n] = f^{-1}({}^{w}_{H}T)$$

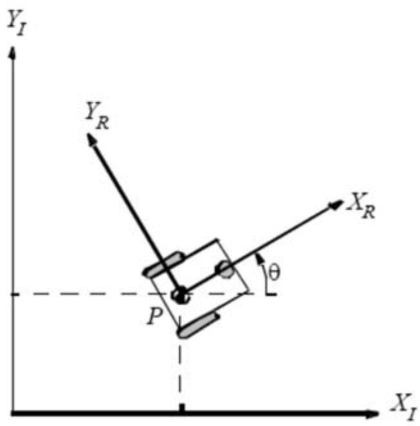


Mobile Robot Kinematics -4

- Representing robot position
  - Initial frame $\{X_I, Y_I\}$
  - Robot frame $\{X_R, Y_R\}$
  - Robot position $\xi_I = [x \quad y \quad \theta]^T$
  - Mapping between two frames

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = R(\theta)\cdot[\dot{x} \quad \dot{y} \quad \dot{\theta}]^T$$

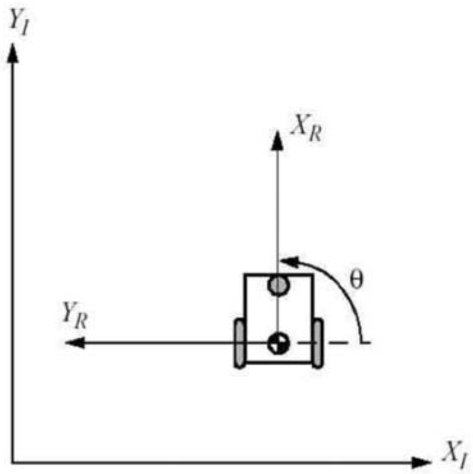
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Mobile Robot Kinematics -5

- Example

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

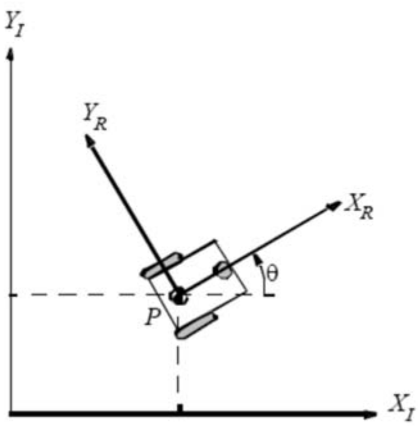


$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

Mobile Robot Kinematics -6

- Forward Kinematic Models
  - Differential drive robot

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\varphi}_r, \dot{\varphi}_l)$$



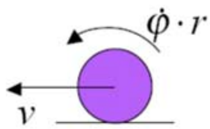
$2l$: distance between wheels
r: wheel radius
$\dot{\varphi}_r$ *and* $\dot{\varphi}_l$: Right and left wheel speeds

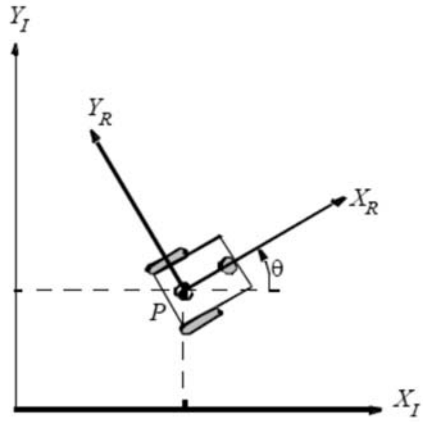
$$= R(\theta)^{-1}\dot{\xi}_R$$

$$= \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{r\dot{\varphi}_r}{2} + \frac{r\dot{\varphi}_l}{2} \\ 0 \\ \frac{r\dot{\varphi}_r}{2l} - \frac{r\dot{\varphi}_l}{2l} \end{bmatrix}$$

Wheel Kinematic Constraints -1

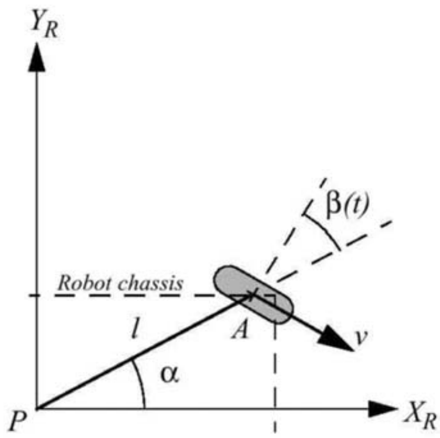
- Assumptions
  - Movement on a horizontal plane
  - Point contact of the wheels
  - Wheels not deformable
  - Pure rolling
  - v = 0 at contact point
  - No slipping, skidding or sliding
  - No friction for rotation around contact point
  - Steering axes orthogonal to the surface
  - Wheels connected by rigid frame (chassis)

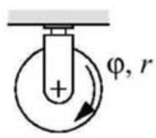




Wheel Kinematic Constraints -2

- Fixed Standard Wheel
  - β is fixed
- Steered Standard Wheel
  - β changes with time





P: reference point on the robot

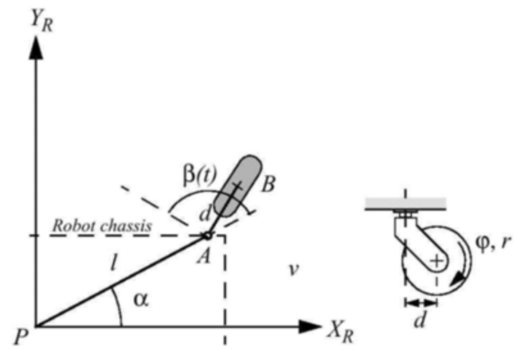
$$[sin(\alpha+\beta) \quad -cos(\alpha+\beta) \quad (-l)cos\beta]R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$=\dot{\xi}_R$$

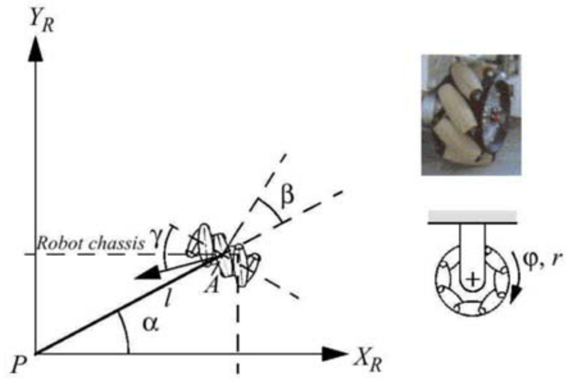
$$[cos(\alpha+\beta) \quad sin(\alpha+\beta) \quad lsin\beta]R(\theta)\dot{\xi}_I = 0$$

Wheel Kinematic Constraints -3

- Castor Wheel

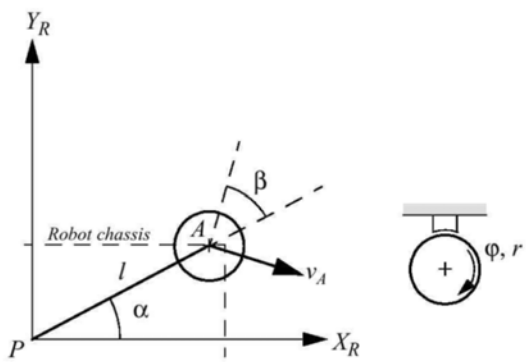


- Swedish Wheel



Wheel Kinematic Constraints -4

- Spherical Wheel



Wheel Kinematic Constraints -5

- Given a robot with M wheels
  - Only fixed and steerable standard wheels impose constraints
  - Others impose zero constraints
- Suppose we have a total of $N = N_f + N_s$ standard wheels
  - Equations for the constraints in matrix forms
  - Rolling

$$J_1(\beta_s)R(\theta)\dot{\xi}_I - J_2\dot{\varphi} = 0 \qquad \varphi(t) = \begin{bmatrix}\varphi_f(t)\\ \varphi_s(t)\end{bmatrix}_{(N_f+N_s)\times 1} \qquad J_1(\beta_s) = \begin{bmatrix}J_{1f}\\ J_{1s}(\beta_s)\end{bmatrix}_{(N_f+N_s)\times 3} \qquad J_2 = diag(r_1 \cdots r_N)$$

  - Sliding

$$C_1(\beta_s)R(\theta)\dot{\xi}_I = 0 \qquad C_1(\beta_s) = \begin{bmatrix}C_{1f}\\ C_{1s}(\beta_s)\end{bmatrix}_{(N_f+N_s)\times 3}$$

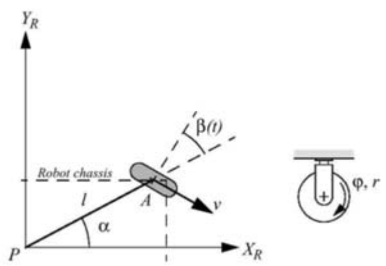
Wheel Kinematic Constraints -6

- Example: Differential Drive Robot
  - Rolling constraint

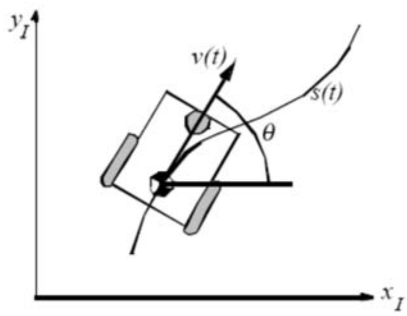
$$[\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad (-l)cos\beta]R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

  - Sliding constraint

$$[cos(\alpha+\beta) \quad sin(\alpha+\beta) \quad lsin\beta]R(\theta)\dot{\xi}_I = 0$$



  - Forward direction $= X_R$
    - Right wheel: $\alpha = -\frac{\pi}{2} \quad \beta = \pi$
    - Left wheel: $\alpha = \frac{\pi}{2} \quad \beta = 0$



Wheel Kinematic Constraints -7

- Constraints

Right wheel J: $[1 \quad 0 \quad l]R(\theta)\dot{\xi}_I - r_r\dot{\varphi}_r = 0$

Left wheel J: $[1 \quad 0 \quad -l]R(\theta)\dot{\xi}_I - r_l\dot{\varphi}_l = 0$

C: $[0 \quad 1 \quad 0]R(\theta)\dot{\xi}_I = 0$

$$\begin{bmatrix} J_1(\beta_s) \\ C_1(\beta_s) \end{bmatrix} R(\theta)\dot{\xi}_I - \begin{bmatrix} J_2\dot{\varphi} \\ 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \end{bmatrix} \\ [0 \quad 1 \quad 0] \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} \begin{bmatrix} r_r & 0 \\ 0 & r_l \end{bmatrix}\begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix} \\ 0 \end{bmatrix}$$

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = R(\theta)^{-1} \begin{bmatrix} \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \end{bmatrix} \\ [0 \quad 1 \quad 0] \end{bmatrix}^{-1} \begin{bmatrix} \begin{bmatrix} r_r & 0 \\ 0 & r_l \end{bmatrix}\begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix} \\ 0 \end{bmatrix}$$

$$\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{2l} & -\frac{1}{2l} & 0 \end{bmatrix} \begin{bmatrix} r_r & 0 \\ 0 & r_l \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r\dot{\varphi}_r}{2} + \frac{r\dot{\varphi}_l}{2} \\ 0 \\ \frac{r\dot{\varphi}_r}{2l} - \frac{r\dot{\varphi}_l}{2l} \end{bmatrix}$$

The same result, formal approach

End

- Questions?