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Sensor Data Fusion for Body State Estimation in a Hexapod Robot with Dynamical Gaits*

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***Abstract*— We report on progress toward a continuous time full 6 DOF translational body state estimator for a hexapod robot executing a jogging gait (with 4 consecutive phases: tripod stance, liftoff transient, aerial, and touchdown transient) on level ground. We use a sequence of dynamical models imported into a standard Kalman Filter to fuse measurements from a novel leg pose sensor and a conventional inertial measurement unit. We implement this estimation procedure on the hexapod robot RHex and evaluate its performance using a visual ground truth measurement system. We also compare the relative performance of different fusion approaches implemented via different model sequences.**

***Index Terms*— Legged Robot, Sensor Fusion, Kalman Filter, Leg Pose Sensor, Inertial Measurement Unit**

I. INTRODUCTION

The hexapod, RHex [1], exhibits unprecedented mobility for a legged autonomous robot [2]. Using an open loop feedforward control strategy, the machine runs at speeds exceeding five body lengths per second on even terrain [3], and negotiates badly broken and unstable surfaces, as well as stairs [4]. Initial empirical studies of controllers relying on cheap and inaccurate sensory feedback cues have resulted in significantly improved performance (inclinometers on slopes [5]; leg touchdown cues over broken terrain [6]) and entirely new behaviors (body pitch sensitive accelerometers for flips [7]; leg touchdown cues for pronking gaits [8]). Theoretical considerations and simulation evidence [9] suggests that the availability of accurate, full body state estimates as well as force interactions with the surrounding environment throughout the stance and aerial phases of locomotion, should confer considerably greater agility still.

However, building a sensor suite that can deliver full body state information — six configuration coordinates together with their six time derivatives — at data rates relevant to motor control ($\sim$ 1kHz) remains a challenging problem in legged robotics because of the constraints upon onboard instrumentation combined with extreme variations in operating regime. Recently, we have introduced a novel leg configuration-based full body pose estimator (hereafter referred to as the "leg pose sensor") for a hexapod robot in tripod stance [10]. In walking gaits with no aerial phase, complete 6 degree of freedom (DOF) body pose in continuous time can easily be extended from the above tripod-stance body pose in principle from a purely kinematic model [11] without velocity state estimation. In contrast, an alternating tripod runner experiencing significant aerial phases (with the concomitant touchdown/liftoff transients) [1] would seem to require full body state estimation — both velocity and configuration information. In order to build the required estimators, of course, the sensor suite must incorporate enough information to allow the reconstruction of full state from the record of past measurement filled in by some dynamical model.

During stance, complete 12 dimensional continuous time body state estimates can be computed from the leg pose sensor by means of direct pose measurement and recourse to online differentiation. Absent any other available sensors, these stance state estimates may be carried through the transient and flight phases only by the adoption of some dynamical prediction model. Although the RHex leg pose sensor delivers accurate high bandwidth body pose estimates during stance (potentially marred by drift effects resulting from toe slippage [11]), overall performance throughout a complete stride is limited by inaccuracies in the transient phase models and the deleterious effects of online differentiation. In contrast, an inertial measurement unit (IMU) continuously delivers derivative (typically linear acceleration and angular velocity) information over all phases of a stride, degraded by saturation and drift effects in the physical sensor that can dramatically reduce the accuracy of the resulting integrated position estimates. Their complementary strengths and weaknesses motivate us to seek better body state estimation than either could afford alone by fusion of the leg pose sensor and IMU together. Therefore, in this paper we join to the leg pose sensor a complete rigid body IMU (3 DOF rate gyro and 3 DOF semiconductor accelerometer) and compare the performance of a few alternative dynamical models in fusion

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[1]Note that hexapedal running gaits need not entail an aerial phase to be "dynamical" in the sense of requiring careful management of kinetic energy to insure balance and steady progress [12]. However, RHex develops its greatest energy efficiency and highest speeds in gaits with long aerial phases, hence, in this paper, we focus our empirical tests on a "jogging" gait with an aerial phase exceeding 25% of the complete stride. By "touchdown" and "liftoff" transients, we refer to intermediate configurations where some number of legs fewer than three are in ground contact.

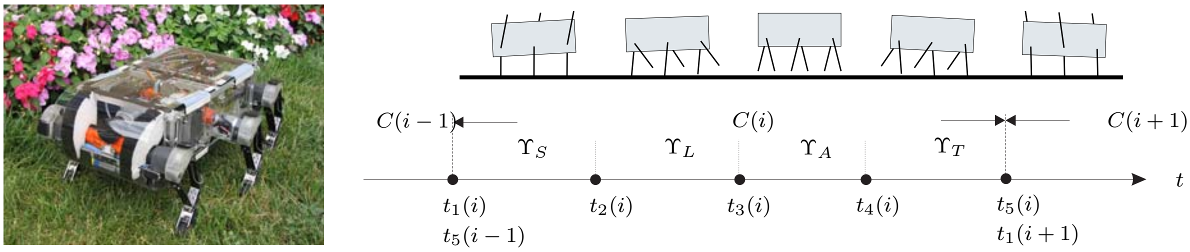

Fig. 1. Left: RHex with 4-bar leg equipped with strain-based leg configuration sensor stands on the meadow; Right: Four consecutive intervals during the i^{th} jogging stride, $C(i)$.

algorithms that deliver translational body state estimates throughout the entire stride in steady state hexapod running. This represents an important first step in a general full (12 dimensional) body state estimator we are presently developing by fusing orientation data and importing more accurate physical-based models for each phase.

The idea of sensor fusion has spread widely within the mobile robotics community, largely for application to wheeled vehicles, including pure inertial navigation systems (INS) with Kalman Filtering [13], INS/GPS fusion, sensor fault detection [14], model selection [15], and INS/vision fusion [16]. However, for legged robots, we have found only a very few accounts of sensor fusion, addressing such problems as sonar-based localization [17] and multilayered-decision algorithms [18]. Nowhere in the databases we have searched[2] have we come across any paper related to full body state estimation in legged robots by sensor data fusion of INS with other sensory modalities. In summary, we find no prior statement nor solution to the problem posed by this paper: the fusion of leg pose and IMU sensor data in a dynamical gait.

Section II introduces notation and illustrates the nature of the "jogging" gait we will study here, followed by a description of algorithm of body state from two independent sensing sources. Section III describes the various dynamical models in each phase of a stride we will build into our statistical filters. Section IV examines the accuracy of the resulting body state estimator implemented on RHex pictured in Figure 1 (Left), followed by a brief conclusion in Section V.

II. Body State from Two Independent Sensing Systems

In this section, we first illustrate the dynamic locomotion in a hexapod robot, followed by a brief review on the structure of the standard Kalman Filter (KF) as a means of establishing notational conventions . Next, we introduce the methodology to obtain full 6 DOF body state in translational motion from two independent sensing sources (leg pose sensor and IMU) without any fusion as the baseline comparison.

A. Dynamical Locomotion (Jogging Gait)

Determining the right dynamical model for the jogging gait promises to be complicated since the physical robot acts as a lagrangian system with 3^6 different models depending on touchdown-stick/touchdown-slip/liftoff conditions on each leg. Without sensing ability to detect toe slippage as well as unknown stability condition under fast switching among large number of models, we simplify that problem by using three models in four successively repeating phases - tripod stance phase, liftoff transient phase, aerial phase, and touchdown transient phase - as a starting point to describe this jogging locomotion and to estimate full body state.

Consider the typical sequence of leg contact conditions occurring during steady state operation in a stable dynamical locomotion depicted in Figure 1 (Right). During the i^{th} stride interval, $C(i){:=}[t_1(i)\ \ t_5(i)]{\subset}R$, a tripod stance interval, $\Upsilon_S(i){:=}[t_1(i)\ \ t_2(i)]$, is succeeded by a period of time when the legs begin to liftoff, $\Upsilon_L(i){:=}[t_2(i)\ \ t_3(i)]$, followed by an interval of aerial flight, $\Upsilon_A(i){:=}[t_3(i)\ \ t_4(i)]$, then touching down through another period of varied leg contacts, $\Upsilon_T(i){:=}[t_4(i)\ \ t_5(i)]$, to the fixed tripod stance interval $\Upsilon_S(i{+}1)$ of the next stride, $C(i{+}1)$. We conceive of the liftoff and touchdown intervals, $\Upsilon_L(i),\Upsilon_T(i)$ as "transients" because they typically exhibit complex sequences of successive leg contacts that reveal little consistent pattern from run to run (or, often, even from stride to stride). In our implementation , the crucial leg contact information required to detect the onset and termination of each of these phases of a stride may be gleaned directly from the individual leg strain-based configuration sensors.

B. Notation Associated with the Kalman Filter

Given a discrete time-invariant plant

$$\mathbf{x}_{k+1} = \Phi\mathbf{x}_k + \Gamma\mathbf{u}_k + \mathbf{w}_k$$

with measurement

$$\mathbf{y}_{k+1} = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k$$

where the process noise $\mathbf{w}_k$ and measurement noise $\mathbf{v}_k$ are white with zero means and covariance defined by $\mathcal{E}[\mathbf{w}_k\mathbf{w}_k^T] = \mathbf{Q}$ and $\mathcal{E}[\mathbf{v}_k\mathbf{v}_k^T] = \mathbf{R}$, a Kalman Filter incorporates two steps: a time update (priori estimate),

$$\begin{aligned}\mathbf{x}_{k+1}^- &= \Phi\mathbf{x}_k^+ + \Gamma\mathbf{u}_k \\ \mathbf{P}_{k+1}^- &= \Phi\mathbf{P}_k^+\Phi^T + \mathbf{Q}\end{aligned} \quad (1)$$

and a measurement update (posteriori estimate)

[2]We have searched the Compendex and IEEExplore data bases using the key words "Legged Robot", "Sensor Fusion".

$$
\begin{array}{rcl}
\mathbf{K}_{k+1} & = & \mathbf{P}_{k+1}^{-}\mathbf{H}^T(\mathbf{H}\mathbf{P}_{k+1}^{-}\mathbf{H}^T+\mathbf{R})^{-1} \\
\mathbf{x}_{k+1}^{+} & = & \mathbf{x}_{k+1}^{-}+\mathbf{K}_{k+1}(\mathbf{z}_k-\mathbf{H}\mathbf{x}_{k+1}^{-}) \\
\mathbf{P}_{k+1}^{+} & = & \mathbf{P}_{k+1}^{-}-\mathbf{K}_{k+1}\mathbf{H}\mathbf{P}_{k+1}^{-}
\end{array} \quad (2)
$$

where $\mathbf{P}$ is error covariance matrix and $\mathbf{K}$ is the so called Kalman gain. Upon initializing the value of state, $\mathbf{x}_0$, and error covariance matrix, $\mathbf{P}_0$, the Kalman Filter continuously delivers the best state "optimal" estimates by consecutively performing these two updates at each time stamp. In practical implementation $\mathbf{w}$ is determined by selection of model [19] and $\mathbf{v}$ is based on experimental noise measurement.

C. Body State from Leg Pose Sensor

During tripod stance phase, $\Upsilon_{S(i)}$, the generalized leg pose sensor (requiring information of toe position with respect to center of mass (COM)) delivers 3 DOF COM translation data with respect to that of initial touchdown moment, $t_1(i)$, detailed in [11]. The velocity state results by differentiating the displacement information and also provides take-off velocity for the following phases, where we adopt a ballistic model for the aerial phase and a constant velocity model for each transient phase (rather than the alternative constant acceleration model) due to significant noise effects in differentiating to obtain takeoff acceleration. These models have a six dimensional state and form the basis for the body translational estimates.

D. Body State from Inertial Measurement Unit

Unlike the leg pose sensor which delivers measurement only during tripod stance phase, an IMU continually delivers measurements across all phases within a stride. Translational body state can be obtained by direct integration and double integration from acceleration data in world coordinates derived from strapdown accelerometer data with orientation compensation made by gyro.

The standard way to apply an Kalman Filter in an "IMU only" navigation system is to use a constant acceleration model as plant, Φ_{IMU}, in the time update and use accelerometer data in the measurement update [13] where

$$
\Phi_{IMU} = \begin{bmatrix} 1 & DT & \frac{1}{2}DT^2 \\ 0 & 1 & DT \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{H}_{IMU} = [0\,0\,1] \quad (3)
$$

and DT is the time difference between measurement[3].

III. Fusion Algorithm for Body State Estimation

In general, sensor selection is strongly constrained by tradeoffs related to hardware issues of space, complexity, cost, performance, and reliability. Once the sensor is chosen, the only room remaining for improvement arises from choice of model/algorithm to achieve better estimation performance. In this paper we focus on comparing the fusion performance of independent sensing source and evaluating the consequences of importing different models into the Kalman Filter.

As discussed in Section II-A, simplifying jogging locomotion by three different models fitting into 4 consecutive phases in one stride significantly reduces the complexity of modeling work. However, selecting correct combination of models for a given succession of phases within a stride remains a challenge. In this initial effort, we have chosen to work with the simplest possible models that decouple the (physically coupled) high dimensional mechanism through reliance on a few basic 1 DOF linear dynamical plants. In a similar spirit, we adopt a "hard" switch (discontinuously reset initial state) between models with continuously passing the initial conditions (state and model covariance), and assume the availability independent, accurate switch timing cues. In future work, we will take a more formal point of view and seek to implement switching procedures based upon the analysis of multiple hybrid models against which this preliminary inquiry may be compared.

We choose a constant acceleration model as one of the models in all three phases due to its straightforward interpretation as the basic model for integration in state equations. In the tripod phase we also try to deploy IMU measurement in prediction equation to form IMU integration model consequently reducing the prediction error in the Kalman Filter. In contrast, in the aerial phase, a ballistic model is clearly motivated by physical first principles. Finally, models of the transient phases amount to guesswork at the present time, and we simply adopt a common sense constant velocity assumption (also used in leg pose sensor).

The details of each model are now listed as follows:

A. Tripod stance phase - constant acceleration model, $\mathbf{G}_{acl}$

The model is the same as shown in (3), and both position measurements from leg pose sensor ($\mathbf{z}_1$) and acceleration measurements from IMU ($\mathbf{z}_2$) are imported in the measurement update:

$$
\Phi_{\mathbf{G}_{acl}} = \begin{bmatrix} 1 & DT & \frac{1}{2}DT^2 \\ 0 & 1 & DT \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{H}_{\mathbf{G}_{acl}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

B. Tripod stance phase - IMU integration model, $\mathbf{G}_{imu}$

We use the IMU measurements acted as "true measurements" as external input ($= \mathbf{u}$) which also plays the role of "true state" in the time update (prediction), and use position measurements from leg pose sensor ($\mathbf{z}_1$) as solo measurements update. In this case, the plant reduces to the constant velocity model:

$$
\Phi_{\mathbf{G}_{imu}} = \begin{bmatrix} 1 & DT & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \Gamma_{\mathbf{G}_{imu}} = \begin{bmatrix} \frac{1}{2}DT^2 \\ DT \\ 1 \end{bmatrix}
$$
$$
\mathbf{H}_{\mathbf{G}_{imu}} = [1\,0\,0]
$$

[3]It has long been remarked in the literature that since its dynamical model is unobservable, the associated Kalman Filter of this "IMU-only" system doesn't guarantee better performance than direct integration. Of course, the naive assumption of white noise and likely inaccurate initial error covariance matrix add to the confusion.

C. Aerial phase - constant acceleration model, $\mathbf{A}_{acl}$

The same as the that in ground phase, but instead of two, only one measurement from IMU ($\mathbf{z}_1$) is available for measurement update: $\Phi_{\mathbf{A}_{acl}} = \Phi_{\mathbf{G}_{acl}}$, $\mathbf{H}_{\mathbf{A}_{acl}} = [0\,0\,1]$.

D. Aerial phase - ballistic model, $\mathbf{A}_{bal}$

The ballistic model is actually a constant velocity model without external input ($\mathbf{u} = 0$) in lateral and forward directions and with gravity as external input ($\mathbf{u} = g$) in vertical direction, and the same as previous one, only one measurement from IMU ($\mathbf{z}_1$) is available for measurement update: $\Phi_{\mathbf{A}_{bal}} = \Phi_{\mathbf{G}_{imu}}$, $\mathbf{H}_{\mathbf{A}_{bal}} = \mathbf{H}_{\mathbf{A}_{acl}}$.

E. Transient phase - constant acceleration model, $\mathbf{T}_{acl}$

Exactly the same as the that in aerial phase: $\Phi_{\mathbf{T}_{acl}} = \Phi_{\mathbf{A}_{acl}}$, $\mathbf{H}_{\mathbf{T}_{acl}} = \mathbf{H}_{\mathbf{A}_{acl}}$.

F. Transient phase - constant velocity model, $\mathbf{T}_{vel}$

The same as ballistic model of aerial phase in lateral and vertical directions (without external input, $\mathbf{u} = 0$): $\Phi_{\mathbf{T}_{vel}} = \Phi_{\mathbf{A}_{bal}}$, $\mathbf{H}_{\mathbf{T}_{vel}} = \mathbf{H}_{\mathbf{A}_{bal}}$.

IV. EXPERIMENT RESULTS

A. Experiment Setup

We have evaluated these estimators experimentally on a version of RHex (50 cm x 20 cm x 30 cm), incorporating the required sensors including: the customized leg pose sensor detailed in [10]; a 3 DOF rate gyro (Fizoptika VG941- 3A) and 3 DOF accelerometer suite (Analog Device ADXL210 ×2) on COM. To assess performance improvements resulting from the fusion of leg pose and IMU data, we have run RHex under the Ground Truth Measurement System (GTMS) the independent visual ground truth measurement system detailed in [20]. This yields another set of 6 DOF translational body state (3 DOF from position measurement and 3 DOF from their derivatives) for comparison. We quantify performance by presenting the standard root mean squared (RMS) error, given by $\chi(p,\hat{p}) := \sqrt{(||p-\hat{p}||_2^2/M)}$ where p represents the state from GTMS; $\hat{p}$ denotes the same state from output of the algorithm; and M is the length of the data.

To establish the baseline for all models, we collect the raw data from sensors and output from GTMS during experimental runs for offline post model performance evaluation. RHex's relatively constrained kinematics precludes the exercise of its yaw degree of freedom (barring intentional excitation of slipping motion on particular toes such as would be required for turning). Therefore, we perform straight-line experimental runs on flat terrain not only to simplify the evaluation process but also to preclude the need for a new turning model, particular to this one robot. we only compare data during stable jogging locomotion after transient from standstill. Table I lists the offline phase timing information on RHex obtained from 10 experimental runs, including length and percentage of time interval of each phase in one complete stride.

TABLE I
EMPIRICAL PHASE RELATIONS IN RHEX JOGGING GAIT

	Tripod Stance mean (std)	Liftoff Transient mean (std)	Aerial Phase mean (std)	Touchdown Transient mean (std)
Average time interval in each phase in one complete stride (ms)				
	96.1 (5.9)	29 (4.9)	55.2 (6.6)	36 (5.5)
Percentage time interval of each phase in one complete stride (%)				
	44.4 (2.73)	13.4 (2.3)	25.5 (3.0)	16.7 (2.6)

B. Performance Evaluation

Table II lists the statistical results (mean and standard deviation from 10 experimental runs) of RMS error between data from output of algorithm and that from GTMS for all 6 DOF translational body state, including COM displacement in lateral (r_x), fore/aft (r_y), vertical (r_z) directions, and their derivatives ($\dot{r_x}$, $\dot{r_y}$, and $\dot{r_z}$). Data associated with leg pose sensor (A), IMU (B), and IMU with KF (C) detailed in Section II-C and Section II-D mean body state derived by only one sensor source without fusion with the other one. After these the table displays results from 8 different fusion algorithms detailed in Section III resulting from the combination of two different models in each of three different phases ($2^3 = 8$) - constant acceleration model ($\mathbf{G}_{acl}$) and IMU integration model ($\mathbf{G}_{imu}$) in tripod stance phase, constant acceleration model ($\mathbf{A}_{acl}$) and ballistic model ($\mathbf{A}_{bal}$) in aerial phase, and constant acceleration/velocity model ($\mathbf{T}_{acl}$/$\mathbf{T}_{vel}$) in transient phase. For convenience in the following discussion, these 8 fusion data traces are numbered from 1 to 8 as the reference index. In this particular setting the leg pose sensor also functions as a touchdown detection sensor for phase switching. Figure 2 shows 3 DOF displacement vs. time from one of the experimental runs, where the trace of each state is broken out in two subplots for ease of view. Each trace represents one state's data over time with a legend according to Table II, including data from independent GTMS (green dot line), leg pose sensor (A, blue solid line), IMU (B, cyan solid line), IMU with KF (C, cyan dashdot line), and 8 fusion results (1-8, red solid line, magenta dashdot line, yellow dash line, and black dot line).

Table II shows the body state from leg pose sensor (A) has good performance in displacement but very poor in velocity due to noisy derivative process. In contrast, that from IMU (B), or IMU with KF (C) which is not much different from (B), has the opposite character - good in velocity but very poor in displacement due to accumulated integration error. This phenomenon also can be checked in Figure 2, especially in r_x where leg pose sensor has better displacement estimates but IMU preserves high frequency components indicating better velocity estimates. These two sensor's dramatically different characteristics serves as the fundamental motivation for fusing sensor data for better state estimation. We also observe that the fusion data, no matter which one we choose among 8, have better performance in comparison to the data from single sensor source for most of the state (but not quite all states).

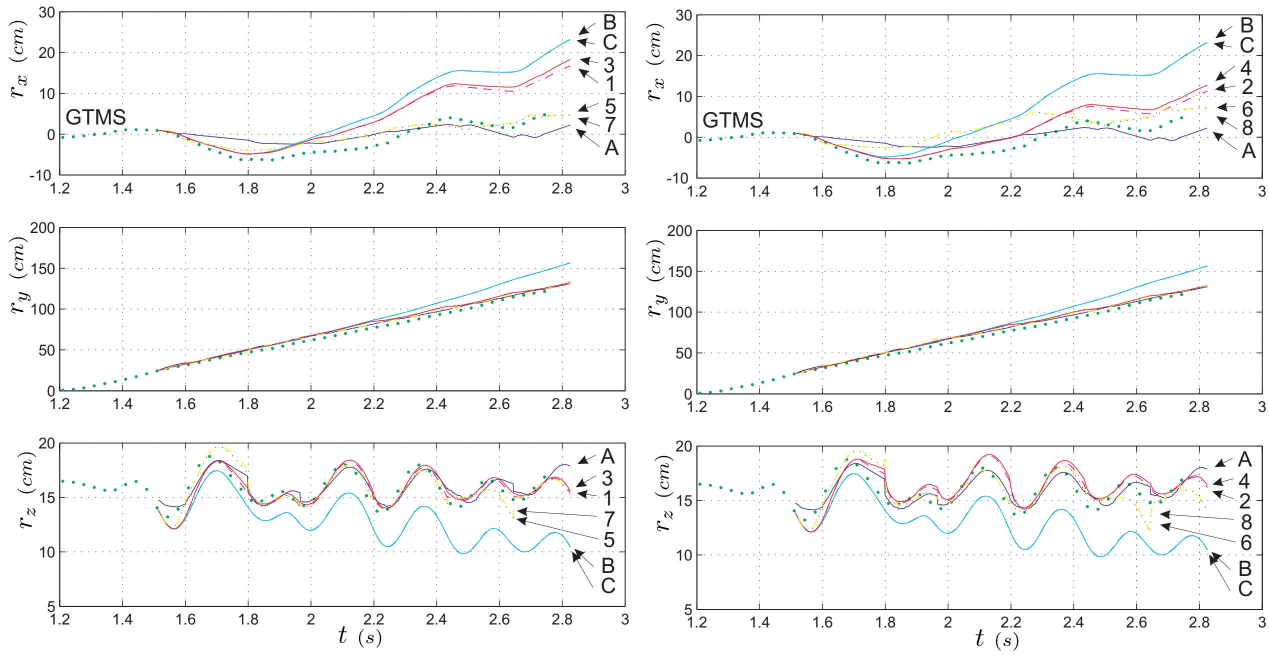

Fig. 2. Translational COM displacement (lateral (r_x), fore/aft (r_y), and vertical (r_z)) measured by GTMS (green dot line), leg pose sensor (A, blue solid line), IMU (B, cyan solid line), IMU with KF (C, cyan dashdot line), and 8 fusion results (1-8, red solid line, magenta dashdot line, yellow dash line, and black dot line) according to Table II.

This is understandable since Kalman Filter "blends" the performance of prediction model with that of true measurement correction based on the idealized assumption of white noises and model covariance, and these departures from the ideal likely preclude uniformly improved performance. Nevertheless, even absent these "requirements" Kalman Filter still performs its robustness and merit to deliver better estimates for most of state.

Table II also suggests the way in which different models may affect the overall performance. For hybrid applications like locomotion requiring rapidly switched it is difficult to tease out the performance of each model from the overall performance since that of each successive individual is strongly affected by initial conditions supplied by its predecessor and in some cases the short time duration of a model's validity may preclude error convergence. However, from the ultimate results still exhibit "trends" that match physical sense and mathematical expectations. For example, in aerial phase the ballistic model ($\mathbf{A}_{bal}$) in most cases is better than kinematic model ($\mathbf{A}_{acl}$) (ex: comparing 7 to 5, 3 to 1). In tripod phase kinematic model ($\mathbf{G}_{acl}$) is better than IMU integration model ($\mathbf{G}_{imu}$) in velocity but opposite in displacement because in kinematic model IMU used in measurement update will keep its strong effect over time which helps performance of velocity but also damages that of displacement, resulting in less performance in displacement comparing to IMU integration model which only use leg pose sensor as measurement update. We also observe overall performance is considerably impacted by models chosen in transient phase.

V. CONCLUSION

We have introduced a continuous time full 6 DOF translational body state estimator for a hexapod robot executing a jogging gait (i.e. with a significant aerial phase) on level ground based on a small number of naive models imported into a standard fusion (Kalman Filter) combining measurements from a leg pose sensor and IMU. We have implemented the algorithm on RHex and evaluated the performance with respect to an independent visual ground truth measurement system (GTMS). We observe smaller RMS errors (in most of the state variables) resulting from the fusion algorithm than from those associated with either single sensor source (either leg pose sensor or IMU) alone.

These results bear out intuition. On the one hand, combining the leg pose sensor and IMU data significantly ameliorates the accumulating integrator drift associated with IMU alone. On the other hand, without IMU's complementary data supplements, the leg pose sensor alone isn't able to deliver continuous time full body state estimation, even with simple models adopted for transient and aerial phases. In practical implementation on RHex IMU has stronger advantage in velocity state while the leg pose sensor is a cleaner source of displacement state information in displacement state. It seems clear that this sort of sensor fusion represents a better alternative for obtaining good state estimates than either sensor type alone.

Beyond the extension to orientation states, the obvious next step toward gaining practical utility for this work lies in the area of multiple hybrid model switching: how to swap out one model and replace it with another model during online execution. To do this properly, it will be

TABLE II

RMS ERROR OF BODY STATE ESTIMATION ACCORDING TO DIFFERENT FUSION ALGORITHMS AND MODELS DETAILED IN SECTION III

	Model Type	Body State - Velocity		
		$\dot{r_x}$ (cm/s) mean (std)	$\dot{r_y}$ (cm/s) mean (std)	$\dot{r_z}$ (cm/s) mean (std)
A	Leg Pose Sensor	22.26 (5.26)	21.62 (3.46)	19.43 (14.36)
B	IMU	14.29 (4.04)	24.74 (3.28)	13.21 (6.92)
C	IMU with KF	14.28 (3.97)	24.91 (3.31)	12.42 (4.81)
1	$\mathbf{G}_{acl}$ + $\mathbf{T}_{acl}$ + $\mathbf{A}_{acl}$	12.38 (3.77)	14.23 (2.49)	13.00 (4.46)
2	$\mathbf{G}_{acl}$ + $\mathbf{T}_{vel}$ + $\mathbf{A}_{acl}$	9.46 (5.56)	13.62 (2.55)	13.19 (5.32)
3	$\mathbf{G}_{acl}$ + $\mathbf{T}_{acl}$ + $\mathbf{A}_{bal}$	12.91 (4.44)	14.13 (2.50)	12.31 (4.71)
4	$\mathbf{G}_{acl}$ + $\mathbf{T}_{vel}$ + $\mathbf{A}_{bal}$	10.19 (6.38)	13.55 (2.55)	13.63 (5.35)
5	$\mathbf{G}_{imu}$ + $\mathbf{T}_{acl}$ + $\mathbf{A}_{acl}$	17.15 (3.61)	13.44 (2.07)	14.76 (3.75)
6	$\mathbf{G}_{imu}$ + $\mathbf{T}_{vel}$ + $\mathbf{A}_{acl}$	18.85 (2.39)	19.65 (4.15)	16.75 (4.54)
7	$\mathbf{G}_{imu}$ + $\mathbf{T}_{acl}$ + $\mathbf{A}_{bal}$	16.38 (3.26)	13.38 (2.04)	13.96 (3.91)
8	$\mathbf{G}_{imu}$ + $\mathbf{T}_{vel}$ + $\mathbf{A}_{bal}$	18.20 (2.28)	19.57 (4.17)	16.34 (4.49)
	Model Type	Body State - Displacement		
		r_x (cm) mean (std)	r_y (cm) mean (std)	r_z (cm) mean (std)
A	Leg Pose Sensor	3.85 (1.52)	4.22 (0.80)	0.68 (0.16)
B	IMU	7.49 (3.17)	12.75 (1.78)	1.57 (1.35)
C	IMU with KF	7.54 (3.17)	12.80 (1.81)	1.63 (1.33)
1	$\mathbf{G}_{acl}$ + $\mathbf{T}_{acl}$ + $\mathbf{A}_{acl}$	5.71 (2.64)	5.16 (0.84)	0.62 (0.17)
2	$\mathbf{G}_{acl}$ + $\mathbf{T}_{vel}$ + $\mathbf{A}_{acl}$	4.10 (3.31)	5.08 (0.82)	0.78 (0.27)
3	$\mathbf{G}_{acl}$ + $\mathbf{T}_{acl}$ + $\mathbf{A}_{bal}$	6.16 (2.80)	5.14 (0.84)	0.66 (0.19)
4	$\mathbf{G}_{acl}$ + $\mathbf{T}_{vel}$ + $\mathbf{A}_{bal}$	4.59 (3.47)	5.06 (0.83)	0.90 (0.30)
5	$\mathbf{G}_{imu}$ + $\mathbf{T}_{acl}$ + $\mathbf{A}_{acl}$	2.52 (0.38)	4.32 (0.82)	0.75 (0.18)
6	$\mathbf{G}_{imu}$ + $\mathbf{T}_{vel}$ + $\mathbf{A}_{acl}$	3.53 (1.20)	4.89 (0.97)	0.95 (0.26)
7	$\mathbf{G}_{imu}$ + $\mathbf{T}_{acl}$ + $\mathbf{A}_{bal}$	2.47 (0.35)	4.31 (0.82)	0.73 (0.18)
8	$\mathbf{G}_{imu}$ + $\mathbf{T}_{vel}$ + $\mathbf{A}_{bal}$	3.54 (1.23)	4.89 (0.97)	0.95 (0.25)

necessary to introduce more realistic physical models, such as the Spring loaded inverted pendulum (SLIP) model for the tripod stance phase, and more complex combinations of physically motivated ground contact models during the touchdown and liftoff transient phases.

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