Chap 2 Mathematical Models of Systems

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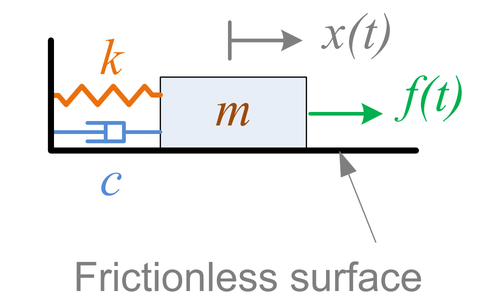
章節內容

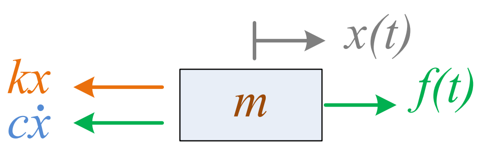
- 2.2 Differential equations of physical systems
- 2.3 Linear approximation of physical systems
- 2.4 The Laplace transform
- 2.5 The transfer function of linear systems
- 2.6 Block diagram models
- 2.7 Signal-flow graph models

Spring-mass-damper (SMD) System

- 要對系統進行量化分析，為系統建立數學模型為首要之務



Free body diagram



Equation of motion (EoM)

$$\sum F_x = m\ddot{x}$$

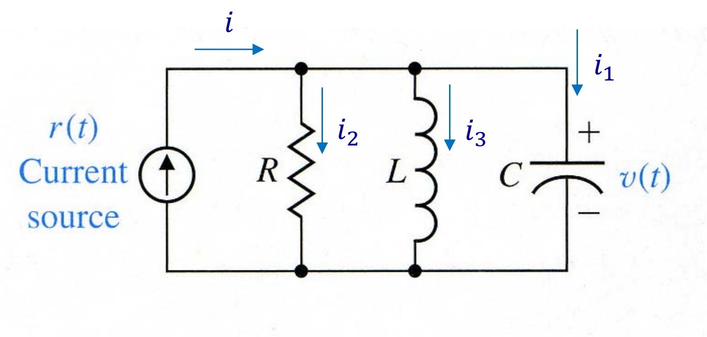
$$f - c\dot{x} - kx = m\ddot{x}$$

$$\Rightarrow \quad m\ddot{x} + c\dot{x} + kx = f$$

Or, set $v = \dot{x}$ and represent EoM

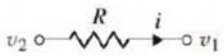
$$m\dot{v} + cv + k\int_0^t v\,dt = f$$

RLC Circuit

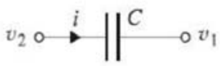


Kirchhoff's law

$$i_1 + i_2 + i_3 = i$$

resistor 

$i = \frac{1}{R} v_{21}$

capacitor 

$i = C \frac{dv_{21}}{dt}$

inductor 

$v_{21} = L \frac{di}{dt}$

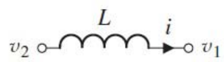
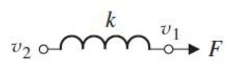
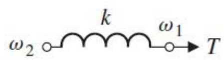
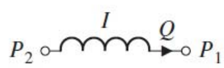
$$c\dot{v} + \frac{1}{R}v + \frac{1}{L}\int_0^t v\,dt = i$$

$$m\dot{v} + cv + k\int_0^t v\,dt = f$$

"Force-current analogy"

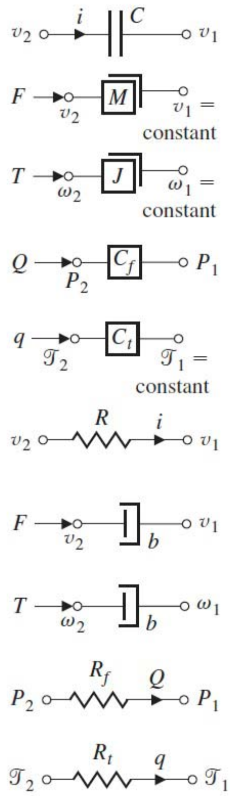
Governing Differential Equations -1

Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	

Governing Differential Equations -2

	Element	Governing equation	Energy E or power $\mathcal{P}$
Capacitive storage	Electrical capacitance	$i = C\dfrac{dv_{21}}{dt}$	$E = \dfrac{1}{2}Cv_{21}{}^2$
	Translational mass	$F = M\dfrac{dv_2}{dt}$	$E = \dfrac{1}{2}Mv_2{}^2$
	Rotational mass	$T = J\dfrac{d\omega_2}{dt}$	$E = \dfrac{1}{2}J\omega_2{}^2$
	Fluid capacitance	$Q = C_f\dfrac{dP_{21}}{dt}$	$E = \dfrac{1}{2}C_fP_{21}{}^2$
	Thermal capacitance	$q = C_t\dfrac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$
Energy dissipators	Electrical resistance	$i = \dfrac{1}{R}v_{21}$	$\mathcal{P} = \dfrac{1}{R}v_{21}{}^2$
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}{}^2$
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}{}^2$
	Fluid resistance	$Q = \dfrac{1}{R_f}P_{21}$	$\mathcal{P} = \dfrac{1}{R_f}P_{21}{}^2$
	Thermal resistance	$q = \dfrac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \dfrac{1}{R_t}\mathcal{T}_{21}$



自動控制（ME3007-01）

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關於

- 系統多為非線性（nonlinear），但在小幅度操作區段內可視為線性（linear）
- Linear system需滿足superposition和homogeneity

Additivity

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$

Scale rule

$x_1(t) \rightarrow y_1(t)$

$\beta x_1(t) \rightarrow \beta y_1(t)$

$$a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t)$$

系統 $y = ax + b$

- 系統本身nonlinear

$$y_1(t) = ax_1(t) + b$$
$$y_2(t) = ax_2(t) + b$$
$$\Rightarrow y_1(t) + y_2(t) = a\big(x_1(t) + x_2(t)\big) + 2b$$
$$\neq a\big(x_1(t) + x_2(t)\big) + b$$

- 但若僅考慮small perturbation，則為linear

$$x(t) = x^* + \Delta x(t) \qquad y^* = ax^* + b \quad \textit{equilibrium point}$$
$$y(t) = y^* + \Delta y(t)$$

$$y^* + \Delta y(t) = a\big(x^* + \Delta \mathrm{x}(t)\big) + b$$

$$\Rightarrow \Delta y(t) = a\Delta x(t)$$ Linear!

建立非線性方程式的線性模型

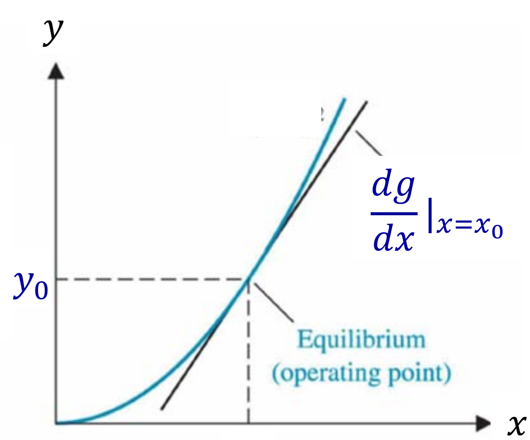
- 利用Taylor series expansion

$$y = g(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

$$a_n = \frac{1}{n!} g^{(n)}(x)\Big|_{x=x_0}$$

x_0: *operating point*

$y_0 = g(x_0) = a_0$



$$y = a_0 + a_1(x - x_0) + \sout{a_2(x - x_0)^2 + \cdots}$$

$| \rightarrow high - order\ terms$ 忽略！

$$y - y_0 = \frac{dg}{dx}\Big|_{x=x_0}(x - x_0)$$

$$\Rightarrow \Delta y = \frac{dg}{dx}\Big|_{x=x_0} \Delta x$$

Example: Nonlinear Spring

- $F = kx^2$ $operating\ point\ x = 1$

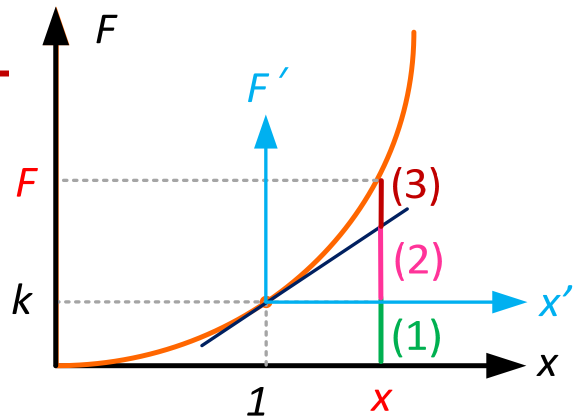
$$a_0 = k \quad a_1 = 2kx\Big|_{x=1} = 2k \quad a_2 = \frac{1}{2}2k\Big|_{x=1} = k$$

$$F = \underbrace{k}_{(1)} + \underbrace{2k(x-1)}_{(2)} + \underbrace{k(x-1)^2}_{(3)}$$

linear terms ← | *"error"*

$$F - k = 2k(x-1)$$

⇒ $\Delta F = 2k\Delta x$



P.S. 上述$purturbation$的方式，也可視為轉換座標來看

$$F' = F - k$$
$$x' = x - 1$$
⇒ $F' = 2kx'$

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章節內容

Laplace Transform定義與特性

- Laplace transform

$$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st} dt$$

- Inverse Laplace transform

$$\mathcal{L}^{-1}[F(s)] = f(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)\, e^{st} ds$$

- P.S. s可視為differential operator

$$s \equiv \frac{d}{dt} \qquad \frac{1}{s} \equiv \int_{0^-}^{t} dt$$

- 僅適用於linear system
- Function f(t)需滿足下方條件始可視為transformable

$\int_{0^-}^{\infty} |f(t)|\, e^{-\sigma_1 t} dt < \infty$ for some real positive σ_1

Laplace Transform Pairs -1

Table 2.3 Important Laplace Transform Pairs

f(t)	*F(s)*
Step function, $u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}}$
$f^{(k)}(t) = \frac{d^k f(t)}{dt^k}$	$s^k F(s) - s^{k-1} f(0^-) - s^{k-2} f'(0^-) - \ldots - f^{(k-1)}(0^-)$

課堂中常用到：

$$\mathcal{L}[\dot{f}(t)] = sF(s) - f(0^-)$$

$$\mathcal{L}[\ddot{f}(t)] = s^2F(s) - sf(0^-) - f'(0^-)$$

Laplace Transform Pairs -2

$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(s)}{s} + \frac{1}{s}\int_{-\infty}^{0} f(t)\, dt$
Impulse function $\delta(t)$	1
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{1}{\omega}[(\alpha-a)^2+\omega^2]^{1/2} e^{-at} \sin(\omega t+\phi)$, $\phi = \tan^{-1}\frac{\omega}{\alpha-a}$	$\frac{s+\alpha}{(s+a)^2+\omega^2}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t, \zeta < 1$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
$\frac{1}{a^2+\omega^2} + \frac{1}{\omega\sqrt{a^2+\omega^2}} e^{-at} \sin(\omega t - \phi)$, $\phi = \tan^{-1}\frac{\omega}{-a}$	$\frac{1}{s[(s+a)^2+\omega^2]}$
$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t + \phi\right)$, $\phi = \cos^{-1}\zeta, \zeta < 1$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
$\frac{\alpha}{a^2+\omega^2} + \frac{1}{\omega}\left[\frac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{1/2} e^{-at} \sin(\omega t+\phi)$. $\phi = \tan^{-1}\frac{\omega}{\alpha-a} - \tan^{-1}\frac{\omega}{-a}$	$\frac{s+\alpha}{s[(s+a)^2+\omega^2]}$

以Laplace Transformation解ODE

- 不需分兩次運算，可同時處理 homogeneous solution和particular solution

- 將differential轉換到algebraic下處理

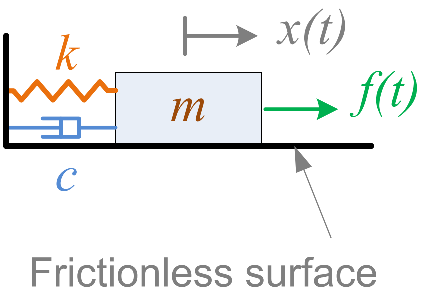
Differential equations (time domain) $\xrightarrow{\mathcal{L}}$ Algebraic equations (s domain) $\xrightarrow{\mathcal{L}^{-1}}$ Solution (time domain)

Differential equations (time domain) ↓ Solution

State-space方法

工數課程所教其他方法等

Example: SMD System - 1



$$m\ddot{x} + c\dot{x} + kx = f$$

$$x(0^-) = x_0 \quad \dot{x}(0^-) = \dot{x}_0$$

$$\downarrow \mathcal{L}$$

$$m(s^2X(s) - sx_0 - \dot{x}_0) + c(sX(s) - x_0) + kX(s) = F(s)$$

$$\Rightarrow X(s) = \frac{(ms + c)x_0 + m\dot{x}_0}{ms^2 + cs + k} + \frac{F(s)}{ms^2 + cs + k}$$

$set\ \ 2\xi\omega_n = \frac{c}{m} \quad \omega_n^2 = \frac{k}{m}$

ξ: damping ratio

ω_n: natural frequency

$$X(s) = \frac{(s + 2\xi\omega_n)x_0 + \dot{x}_0 + \frac{F(s)}{m}}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Example: SMD System - 2

Assume $m = 1 \;\; c = 4 \;\; k = 3 \quad f(t) = 2\ (step\ input)$

$x_0 = 1 \;\; \dot{x}_0 = 0$

$\ddot{x} + 4\dot{x} + 3x = 2$

$\downarrow \mathcal{L}$

解法：
以等式求係數
Heaviside's Formula

$$X(s) = \frac{s+4}{s^2+4s+3} + \frac{\frac{2}{s}}{s^2+4s+3}$$

$$= \left(\frac{k_1}{s+1} + \frac{k_2}{s+3}\right) + \left(\frac{k_3}{s+1} + \frac{k_4}{s+3} + \frac{k_5}{s}\right)$$

針對左側

$$k_1 = \frac{s+4}{(s+1)(s+3)}(s+1)\Big|_{s=-1} = \frac{3}{2}$$

$$k_2 = \frac{s+4}{(s+1)(s+3)}(s+3)\Big|_{s=-3} = -\frac{1}{2}$$

Example: SMD System - 3

針對右側，同理

$$k_3 = \frac{2}{s(s+1)(s+3)}(s+1)\Big|_{s=-1} = -1 \qquad k_4 = \frac{1}{3} \qquad k_5 = \frac{2}{3}$$

$\mathcal{L}^{-1}$

$$x(\mathrm{t}) = \left(\frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t}\right) + \left(-e^{-t} + \frac{1}{3}e^{-3t} + \frac{2}{3}\right)$$

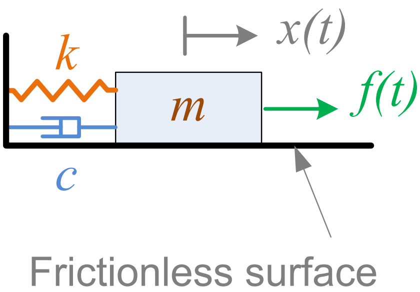
- $k_1\ k_2$ terms: Transient response due to I.C.
- $k_3\ k_4$ terms: Transient response due to f
- k_5 term: Steady response due to f

Note:

from $m\ddot{x} + c\dot{x} + kx = f$

in steady state → $\ddot{x} = 0 \;\; \dot{x} = 0$ ⇒ $x = \frac{f}{k} = \frac{2}{3}$

想一下



- 若SMD系統是垂直放置的，有受到重力的影響，EoM和系統的靜態動態狀態會有什麼變化？

- 假設SMD example中其他參數的數值保持不變，僅將$k = 4$或$k = 5$，系統的解有什麼變化？

一些Terminologies

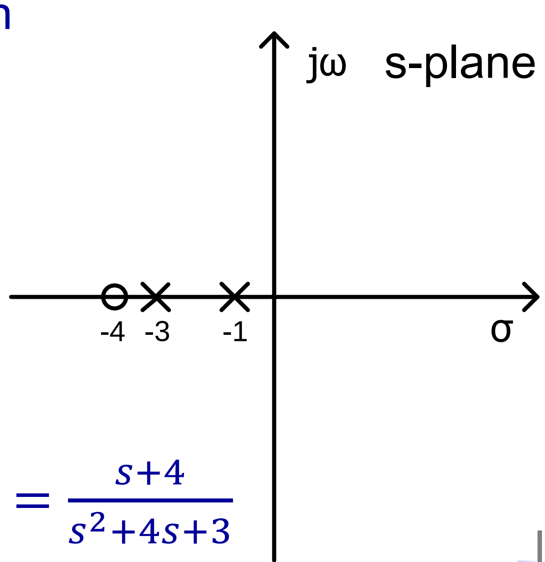
接前頁

$$X(s) = \frac{p(s)}{q(s)}$$

$q(s) = 0$ characteristic equation

$s|_{q(s)=0}$ poles of the system

$s|_{p(s)=0}$ zeros of the system



Ex: $X(s) = \frac{s+4}{s^2+4s+3}$

兩個定理

- Initial value theorem

$$f(0^+) = \lim_{s\to\infty} sF(s)$$

- Final value theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$F(s)$: *no pole on Im − axis or RHP*
can have a simple pole at $s = 0$

P.S. $S - M - D$ *system*

$$X(s) = \frac{s^2 + 4s + 2}{s(s^2 + 4s + 3)}$$

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) = \frac{2}{3}$$

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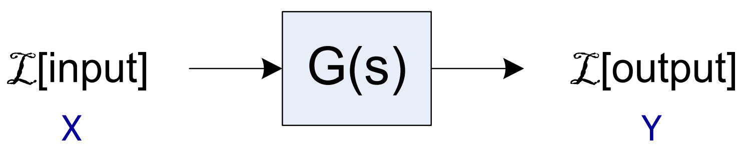
Transfer Function

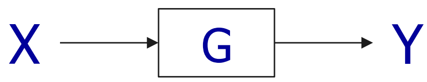
- 定義

$$T.F. = G(s) = \frac{\mathcal{L}\,\{output\}}{\mathcal{L}\,\{input\}} = \frac{Y}{X} \quad with\ initial\ condition\ (I.C.) = 0$$



- 特性
  - 僅適用於linear和stationary（constant parameters）的系統
  - 僅表達系統input和output之間的關係（注意在Laplace的轉換過程中，I.C.s都要設定為0），不似state-space的表達法，也包含系統內部的組成方式（在第三章有進一步的說明）

Examples -1



- Differentiator

$$input = x \qquad output = y = \dot{x}$$

$$G(s) = \frac{\mathcal{L}\{output\}}{\mathcal{L}\{input\}} = \frac{sX}{X} = s$$

- Integrator

$$input = \dot{x} \qquad output = y = x$$

$$G(s) = \frac{\mathcal{L}\{output\}}{\mathcal{L}\{input\}} = \frac{X}{sX} = \frac{1}{s}$$

Examples -2

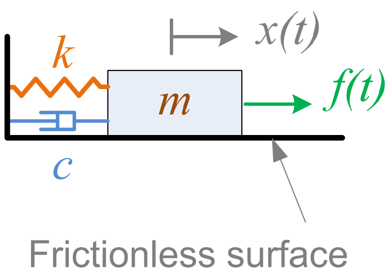
- Spring-mass-damper system

$$m\ddot{x} + c\dot{x} + kx = f$$

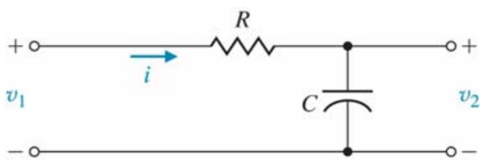
$$input = f \qquad output = x$$

$$G = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$

"two poles"



- RC circuit



$$input = v_1 \qquad output = v_2$$

$$v = Ri \rightarrow V = RI$$

$$i = C\frac{dv}{dt} \rightarrow V = \frac{1}{Cs}I$$

$$G = \frac{V_2}{V_1} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{RCs + 1}$$

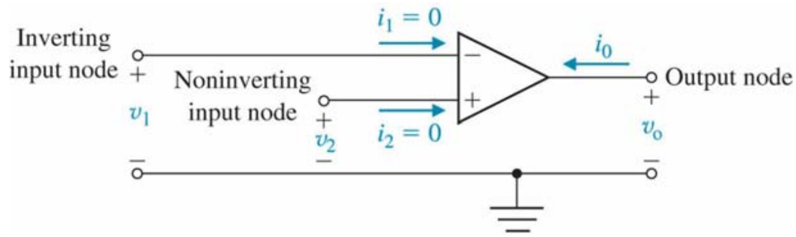
$$\tau = RC = time\ constant$$

Example -3

- OP-amp Golden rules

(1) $i_1 = i_2 = 0$

(2) $v_1 = v_2$

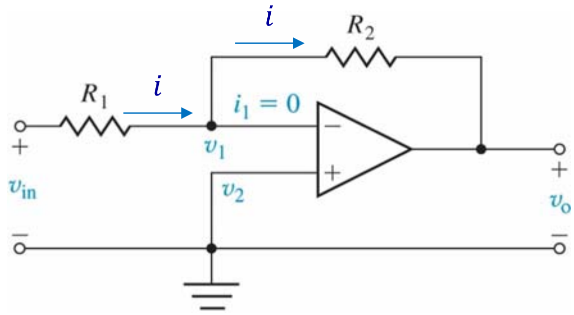


- OP inverting amplifier

$$i = \frac{v_{in} - 0}{R_1}$$

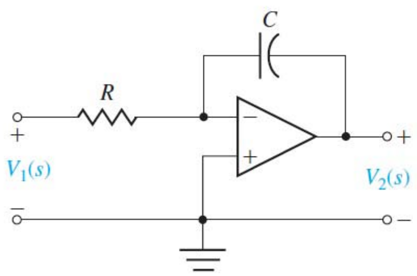
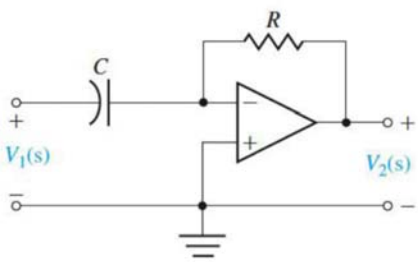
$$v_0 = 0 - R_2 i = -\frac{R_2}{R_1} v_{in}$$

$$\Rightarrow \quad G = \frac{V_0}{V_{in}} = -\frac{R_2}{R_1}$$

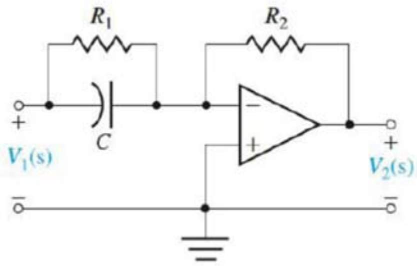


Transfer Functions of OP Circuits -1

Table 2.5 Transfer Functions of Dynamic Elements and Networks

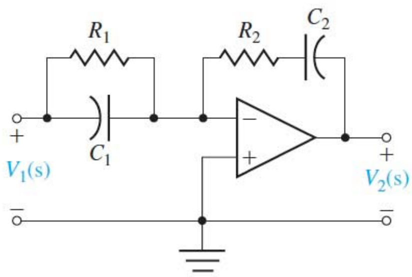
Element or System	G(s)
1. Integrating circuit, filter 	$\frac{V_2(s)}{V_1(s)} = -\frac{1}{RCs}$
2. Differentiating circuit 	$\frac{V_2(s)}{V_1(s)} = -RCs$

3. Differentiating circuit



$$\frac{V_2(s)}{V_1(s)} = -\frac{R_2(R_1Cs + 1)}{R_1}$$

4. Integrating filter



$$\frac{V_2(s)}{V_1(s)} = -\frac{(R_1C_1s + 1)(R_2C_2s + 1)}{R_1C_2s}$$

(continued)

自動控制（ME3007-01）

Chap 2 Mathematical Models of Systems

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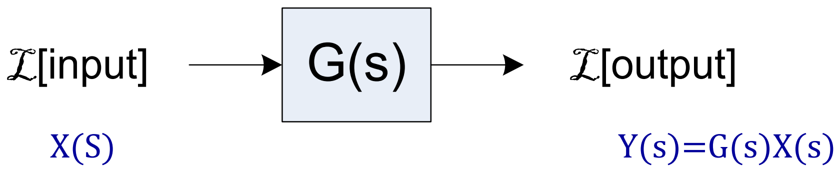
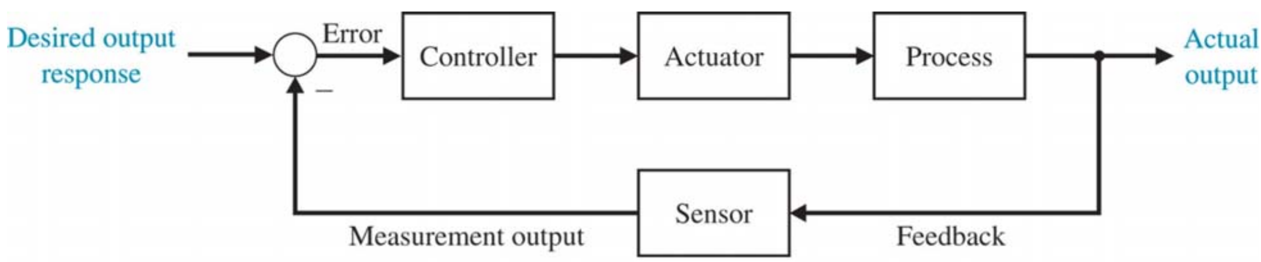
章節內容

Block Diagram

- Transfer function

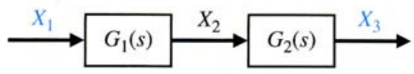
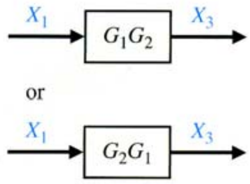
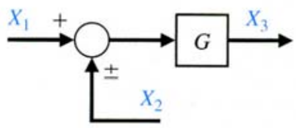
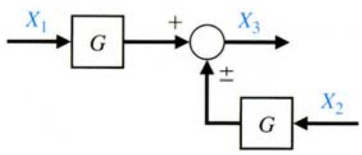
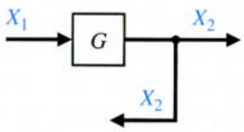
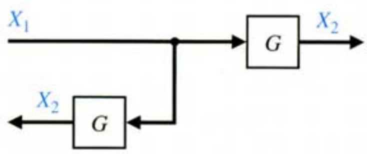


- Block diagram: Representing the relationship of system variables by diagrammatic means

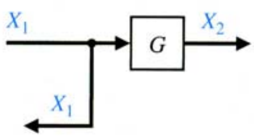
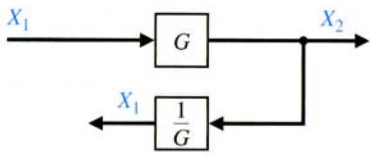
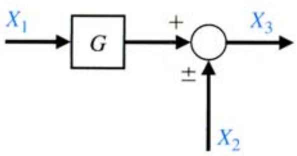
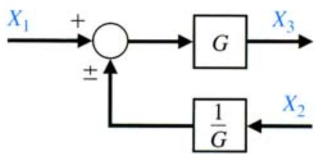
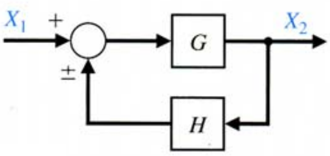
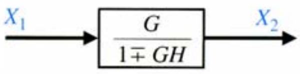


Block Diagram Transformations -1

Table 2.6 Block Diagram Transformations

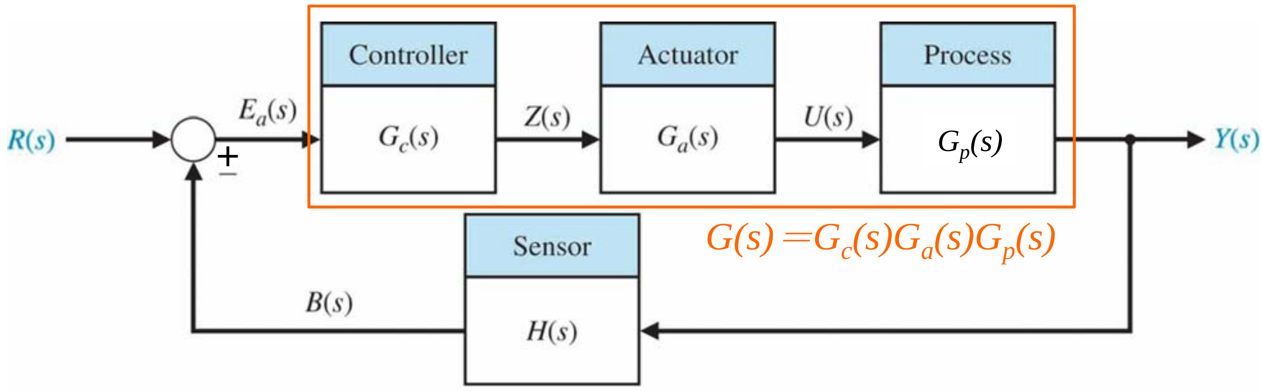
Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade	 $X_3 = G_2 X_2 = G_2(G_1 X_1)$ $= G_2 G_1 X_1$ $= G_1 G_2 X_1$	
2. Moving a summing point behind a block	 $X_3 = G\ (X_1 \pm X_2)$ $= GX_1 \pm GX_2$	
3. Moving a pickoff point ahead of a block		

Block Diagram Transformations -2

Transformation	Original Diagram	Equivalent Diagram
4. Moving a pickoff point behind a block		
5. Moving a summing point ahead of a block		
6. Eliminating a feedback loop		

見下頁

Closed-loop Transfer Function

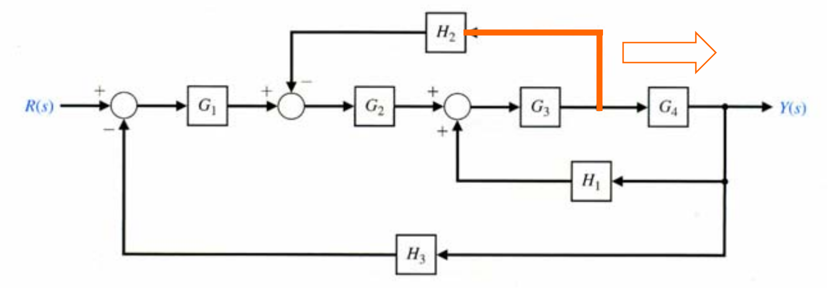


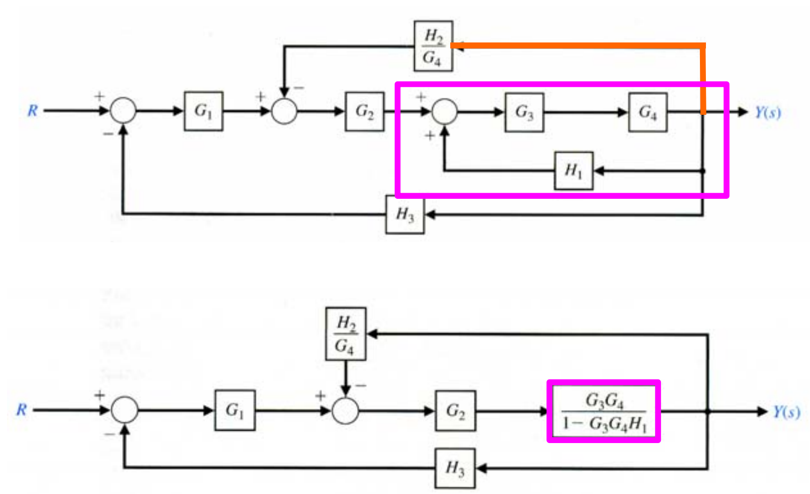
$$Y = GE_a = G(R \pm B) = G(R \pm HY) = GR \pm GHY$$

$$(1 \mp GH)Y = GR$$

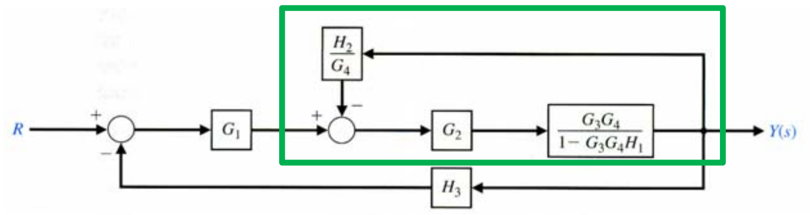
$$\Rightarrow \quad \frac{Y}{R} = \frac{G}{1 \mp GH}$$

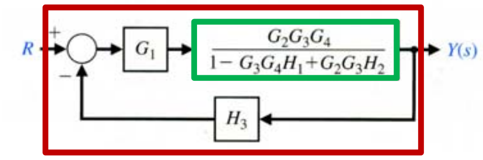
Block Diagram Reduction -1

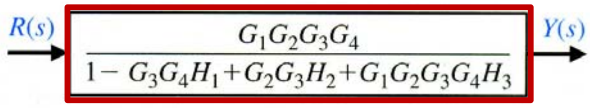


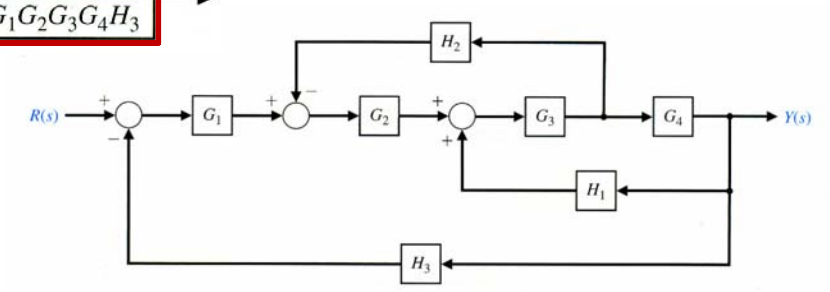


Block Diagram Reduction -2









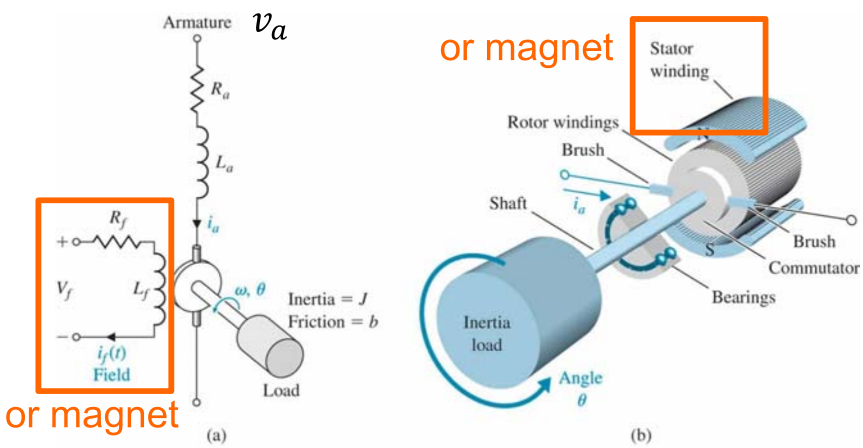
Example: Armature-controlled DC Motor -1

- Mechanical

$$\tau = J\ddot{\theta} + b\dot{\theta} = J\dot{\omega} + b\omega$$

$\downarrow \mathcal{L}$

$$T = J\Theta s^2 + b\Theta s = JWs + bW$$



- Electrical

$$v_a = Ri + L\frac{di}{dt} + \underbrace{k_b\omega}_{\text{Back emf}}$$

$\downarrow \mathcal{L}$

$$V_a = RI + LIs + k_bW$$

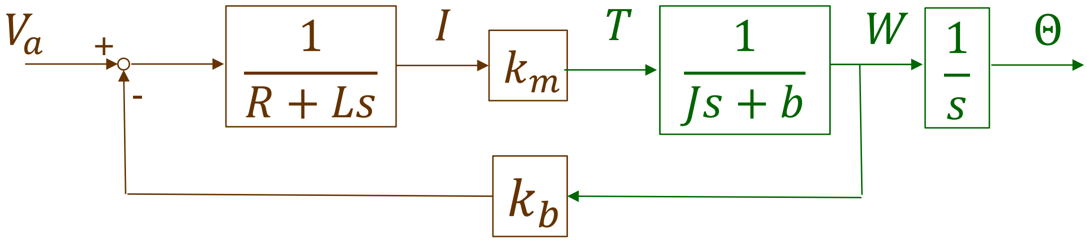
$$I(s) = \frac{V_a(s) - k_bW(s)}{R + Ls}$$

$$\tau = \underbrace{k_m}_{\text{Motor constant}} i$$

$\downarrow \mathcal{L}$

$$T = k_mI = k_m\frac{V_a - k_bW}{R + Ls}$$

Example: Armature-controlled DC Motor -2



$$G = \frac{\Theta}{V_a} = \frac{1}{s}\frac{k_m}{(R+Ls)(Js+b)+k_b k_m}$$

Note: power: mechanical electrical

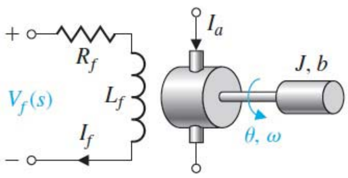
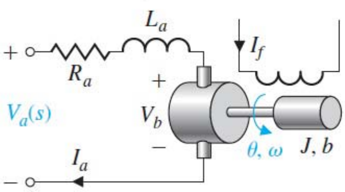
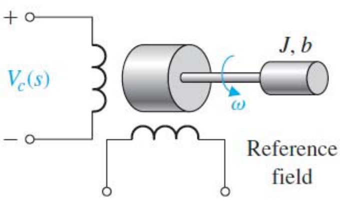
$$\tau \cdot \omega = (k_m i)\omega = (k_b \omega) i$$

$$\Rightarrow k_m = k_b$$

$$unit: \left(\frac{N-m}{A}\right) \quad \left(\frac{V}{rad \cdot s}\right)$$

Transfer Function of Dynamic Components

Table 2.5 ***Continued*** P.S. 課本上還有其他案例

Element or System	G(s)
5. DC motor, field-controlled, rotational actuator 	$\frac{\theta(s)}{V_f(s)} = \frac{K_m}{s(Js+b)(L_f s+R_f)}$
6. DC motor, armature-controlled, rotational actuator 	$\frac{\theta(s)}{V_a(s)} = \frac{K_m}{s[(R_a+L_a s)(Js+b)+K_b K_m]}$
7. AC motor, two-phase control field, rotational actuator 	$\frac{\theta(s)}{V_c(s)} = \frac{K_m}{s(\tau s+1)}$ $\tau = J/(b-m)$ m = slope of linearized torque-speed curve (normally negative)

Chap 2 Mathematical Models of Systems

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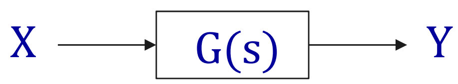
章節內容

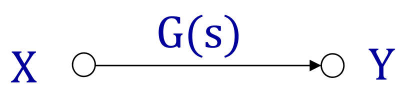
Signal-flow Graph定義 -1

- A graphical representation of a set of linear relations which can derive gain (i.e., T.F.) without any reduction procedure

Block diagram



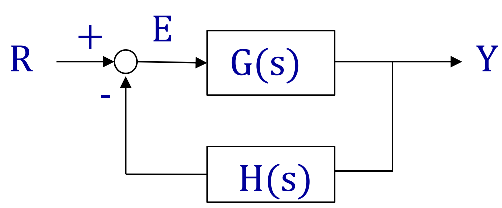
Signal-flow graph



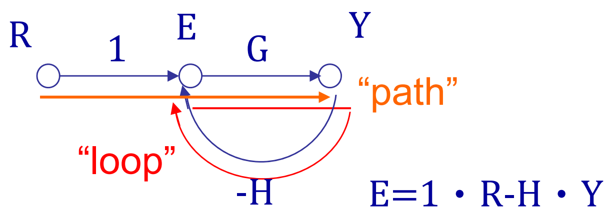
- Definition
  - Node: input/output point or junction
  - Branch: Relating the dependency of an input and an output

Signal-flow Graph定義 -2

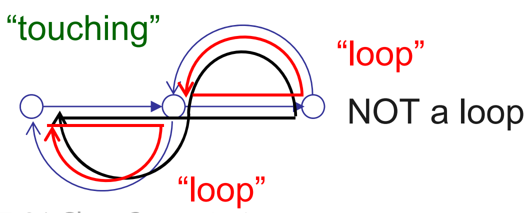
Block diagram

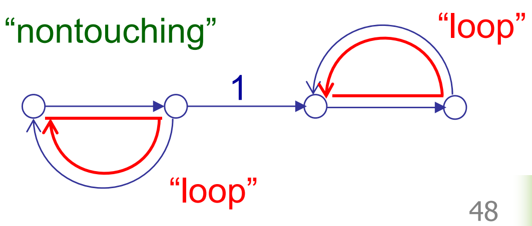


Signal-flow graph



- Path: a branch or a continuous sequence of branches; no node is met more than once (ex: from X to Y, from R to Y)
- Loop: a closed path; no node is met more than once (ex: from E to Y to E)
- Nontouching: two loops do not have common nodes





Mason's Formula

- T.F. = $T_{ij} = \dfrac{\sum_k P_{ijk}\Delta_{ijk}}{\Delta}$

- P_{ijk} = k^{th} path form variable x_i to variable y_j

- Δ = determinant of the graph

$$= 1 - \sum_{n=1}^{N} L_n + \sum_{m=1,g=1}^{M,\vartheta} L_m L_g - \sum L_r L_s L_t + \cdots$$

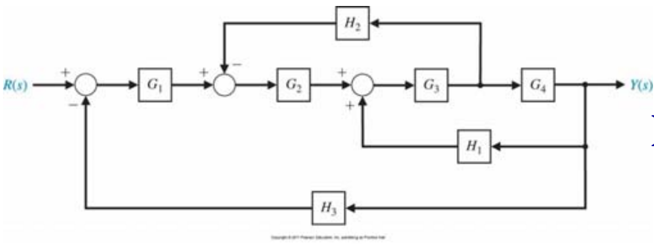
L_x: *loop* "nontouching" "nontouching"

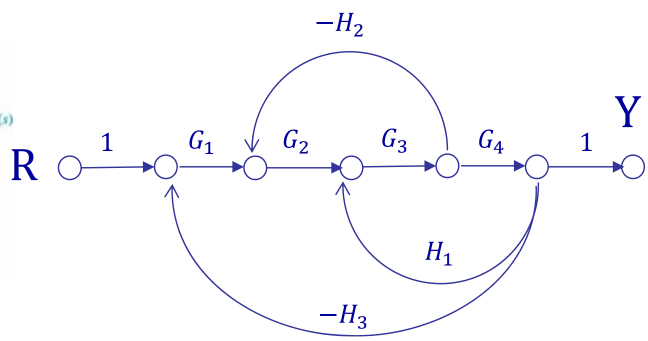
- Δ_{ijk} = cofactor of the path P_{ijk}

$$= \Delta - (\text{any term touching } k^{\text{th}} \text{ path})$$

Example

- Revisit BD reduction problem





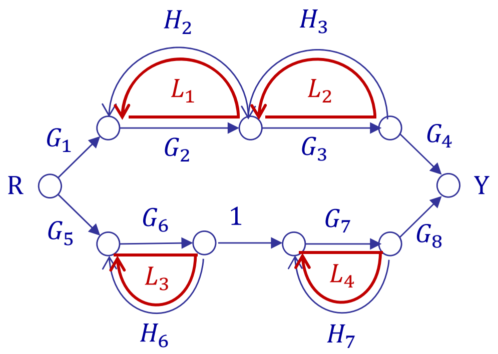
$$P_{RY1} = p_1 = G_1G_2G_3G_4$$

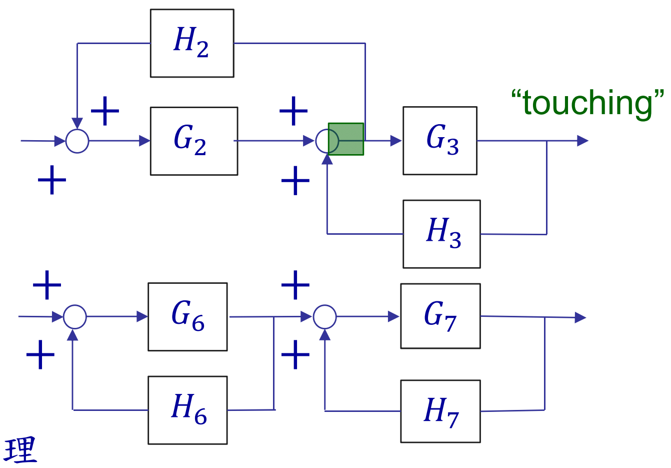
$$\Delta = 1 - (G_3G_4H_1 - G_2G_3H_2 - G_1G_2G_3G_4H_3)$$

$$\Delta_{RY1} = 1 \qquad \text{"all touching"}$$

$$T_{RY} = \frac{P_{RY1}\Delta_{RY1}}{\Delta} = \frac{G_1G_2G_3G_4}{1 - G_3G_4H_1 - G_2G_3H_2 + G_1G_2G_3G_4H_3}$$

Example





Method 1 上下path分開，並聯處理

"nontouching"

$$T = G_1\left(\frac{G_2G_3\cdot 1}{1-L_1-L_2}\right)G_4 + G_5\left(\frac{G_6}{1-L_3}\cdot 1\cdot\frac{G_7}{1-L_4}\right)G_8 = \cdots$$

$$= \frac{G_6G_7}{1-L_3-L_4-L_3L_4}$$

Method 2 整個一起看

$$T = \frac{G_1G_2G_3G_4(1-L_3-L_4+L_3L_4)+G_5G_6G_7G_8(1-L_1-L_2)}{1-(L_1+L_2+L_3+L_4)+(L_1L_3+L_1L_4+L_2L_3+L_2L_4+L_3L_4)-(L_1L_3L_4+L_2L_3L_4)}$$

Matlab – 以SMD系統為例

- 建立Transfer function $G(s) = \dfrac{1}{s^2+4s+3}$
  - 方法一 `>>sys=tf(1,[1 4 3]);`
  - 方法二 `>>s=tf('s'); sys2=1/(s^2+4*s+3)`
- 系統找poles和zeros

```
>>pole(sys)
>>zero(sys)

>>[p,z]=pzmap(sys)
```

P.S. 下列方法也可找poles

```
>>den=[1 4 3]
>>roots(den)
```

- 系統並聯、串聯、回授、step response

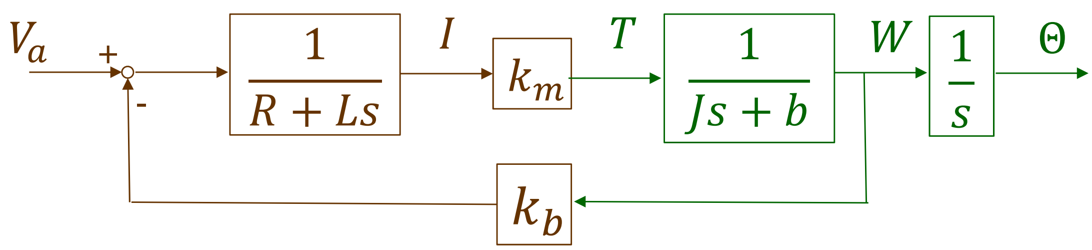
```
>>parallel(sys1,sys2)
>>series(sys1,sys2)
>>feedback(G,H,-1)
>>step(sys)
```

P.S. 對Matlab指令使用方式有疑問可用help功能

Ex. `>>help feedback`

Matlab練習

- 以ppt和課本Section 2.8所教指令，建立Armature-controlled DC motor 例題中各個block，並以「Matlab指令」計算出整個馬達「由電壓v到轉速w」的transfer function G(s)



- 上網查詢小馬達的規格表，帶入馬達參數，給馬達供給定電壓，觀察馬達轉速的反應(step response)

終

- Questions?