Chap 4 Feedback Control System Characteristics

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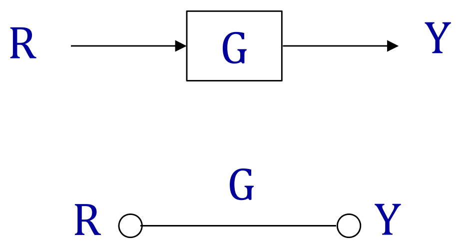
- Error signal analysis
- Sensitivity of control systems to parameter variations
- Disturbance signals in a feedback control system
- Control of the transient response
- Steady-state error
- The cost of feedback

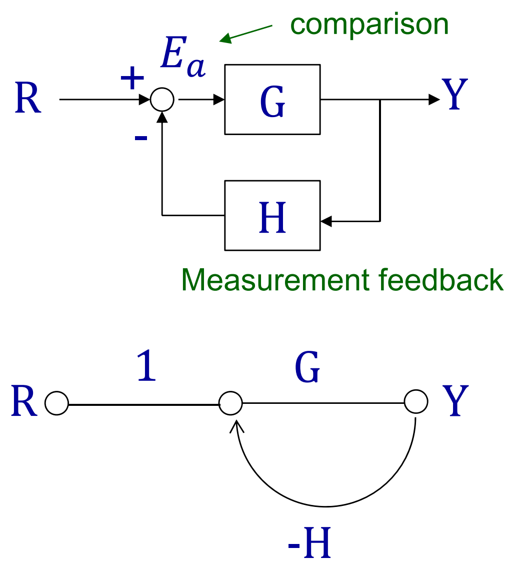
本章敘事架構

- 比較open-loop和closed-loop系統在數項重要系統特性上的表現和差異

Open-loop



Closed-loop



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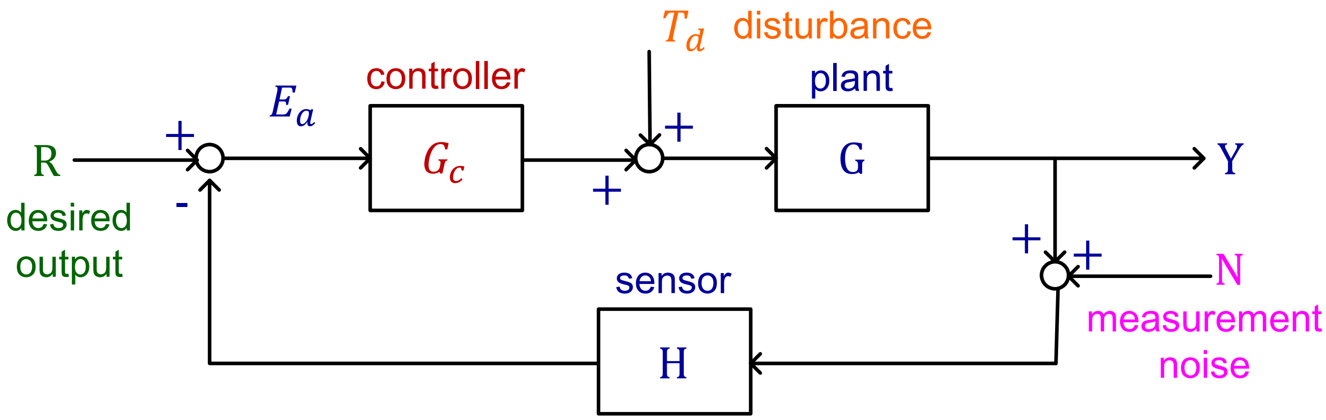
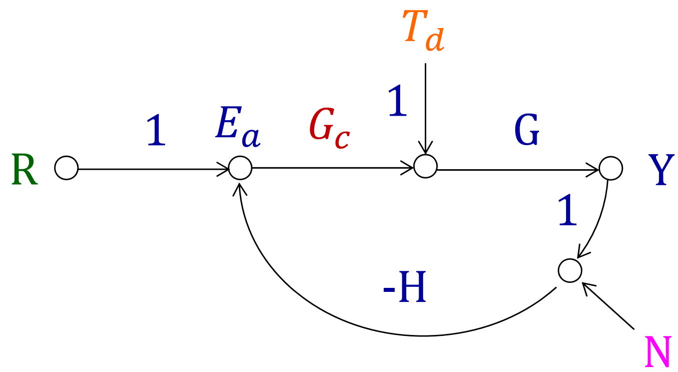
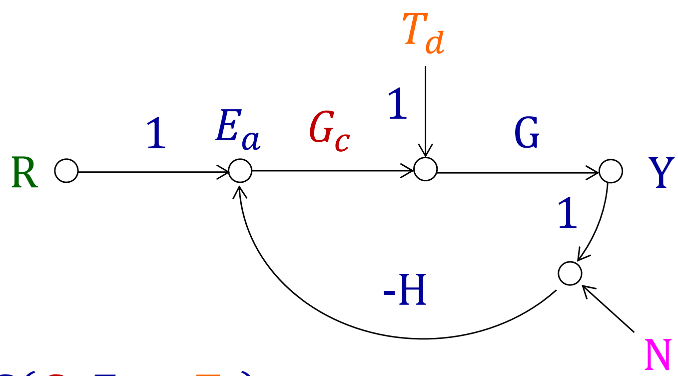
Error Signal Analysis -1





Error Signal Analysis -2



$$Y = G(G_c E_a + T_d)$$

$$= G G_c \big(R - H(N + Y)\big) + G T_d$$

$$= G G_c R - G G_c H N - G G_c H Y + G T_d$$

$$Y = \frac{G_c G}{1 + G_c G H} R + \frac{G}{1 + G_c G H} T_d + \frac{-G_c G H}{1 + G_c G H} N$$

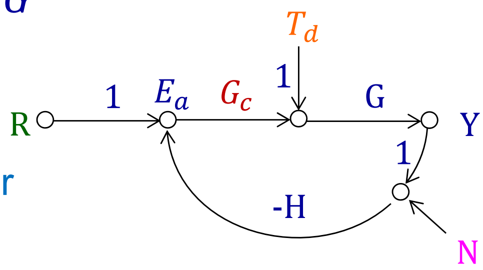
Error Signal Analysis -3

- Consider H(s)=1 (ease of discussion)

$$Y = \frac{G_c G}{1+G_c G} R + \frac{G}{1+G_c G} T_d + \frac{-G_c G}{1+G_c G} N$$

define $L = G_c G$ loop gain

$E = R - Y$ tracking error



$$E = \frac{1}{1+L} R + \frac{-G}{1+L} T_d + \frac{L}{1+L} N$$

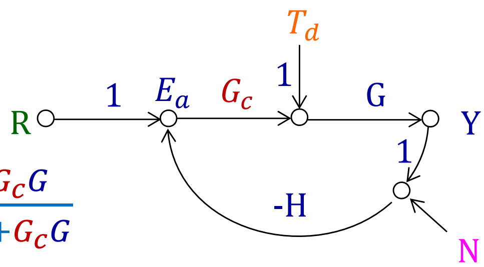
define $S = \dfrac{1}{1+L}$ sensitivity function

$C = \dfrac{L}{1+L}$ complementary sensitivity function

Error Signal Analysis -4

$$E = SR - SGT_d + CN$$

given process (or plant)



prefer: small S=$\dfrac{1}{1+G_c G}$ & C=$\dfrac{G_c G}{1+G_c G}$

reality: S+C=1 (design compromise)

G_c : large to ↓ T_d

small to ↓ N

strategy: Make G_c (s) large at low frequencies

small at high frequencies

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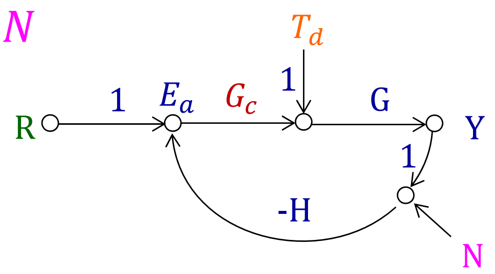
Sensitivity -1

$$Y = \frac{G_c G}{1 + G_c G} R + \frac{G}{1 + G_c G} T_d + \frac{-G_c G}{1 + G_c G} N$$

(P.S. $H(s) = 1$)



$$Y = \frac{G_c G}{1 + G_c G} R \quad (\text{set } T_d = 0 \quad N = 0)$$

$$E = R - Y$$

$$= \frac{1}{1 + G_c G} R$$

$$E + \Delta E = \frac{1}{1 + G_c (G + \Delta G)} R$$

variation in process/plant

$$\Delta E = \frac{-G_c \Delta G}{(1 + G_c G + G_c \Delta G)(1 + G_c G)} R$$

Usually $G_c G \gg G_c \Delta G$ and $G_c G \gg 1$

$$\approx \frac{-G_c \Delta G}{(G_c G)^2} R$$

$$= -\frac{1}{L} \frac{\Delta G}{G} R \quad G_c G = L \uparrow \text{ to let } \Delta E \downarrow$$

So $L = G_c G \uparrow$

$Y \approx R$

$E \downarrow$

Sensitivity -2

T: closed-loop T.F.

$$S_G^T = system\ sensitivity \triangleq \frac{\frac{\Delta T}{T}}{\frac{\Delta G}{G}} = \frac{\Delta T}{\Delta G}\frac{G}{T} = \frac{\partial T}{\partial G}\frac{G}{T}$$

If $G=G(\alpha)$ α: a parameter in G

(chain rule)

$$S_\alpha^T = \frac{\partial T}{\partial \alpha}\frac{\alpha}{T} = \frac{\partial T}{\partial G}\frac{G}{T}\frac{\partial G}{\partial \alpha}\frac{\alpha}{G} = S_G^T S_\alpha^G$$

evaluate effect of α

Sensitivity -3

- Open-loop v.s. Closed-loop

$$R \longrightarrow \boxed{G} \longrightarrow Y$$

Open-loop $S_G^T = 1$ $(\because T = G)$

Closed-loop $T = \dfrac{G_c G}{1 + G_c G}$ (assume $H(s)=1$)

$$S_G^T = \frac{\partial T}{\partial G}\frac{G}{T} = \frac{G_c}{(1+G_cG)^2}\frac{G}{\frac{G_cG}{1+G_cG}}$$

$$= \frac{1}{(1+G_cG)}$$

So $L = G_cG \uparrow \quad S_G^T \downarrow$

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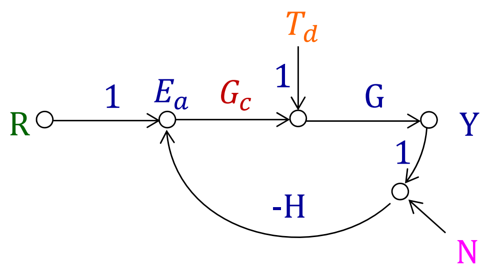
Disturbance -1

- $Y = \frac{G_c G}{1 + G_c G} R + \frac{G}{1 + G_c G} T_d + \frac{-G_c G}{1 + G_c G} N$

$$E = \frac{1}{1 + L} R + \frac{-G}{1 + L} T_d + \frac{L}{1 + L} N$$

(P.S. $H(s) = 1$)

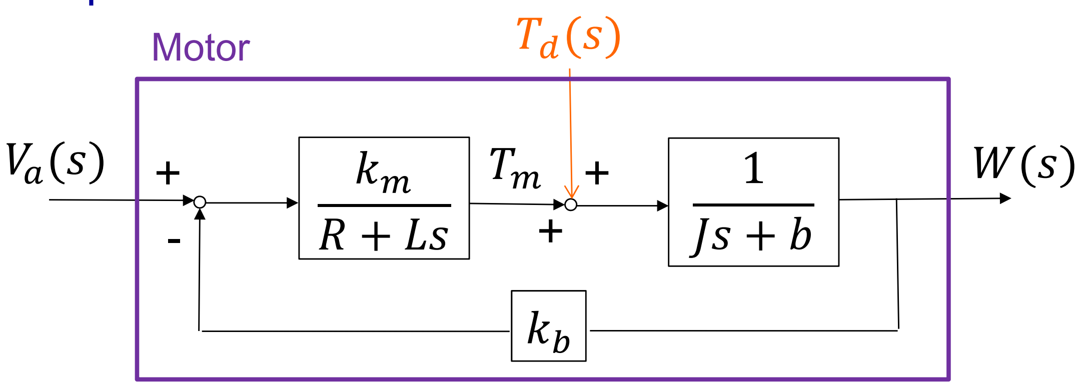


$$E = \frac{-G}{1 + L} T_d \quad (\text{set } R = 0 \quad N = 0)$$

So $L = G_c G \uparrow \quad E \downarrow$

Disturbance -2

- EX: motor speed-control



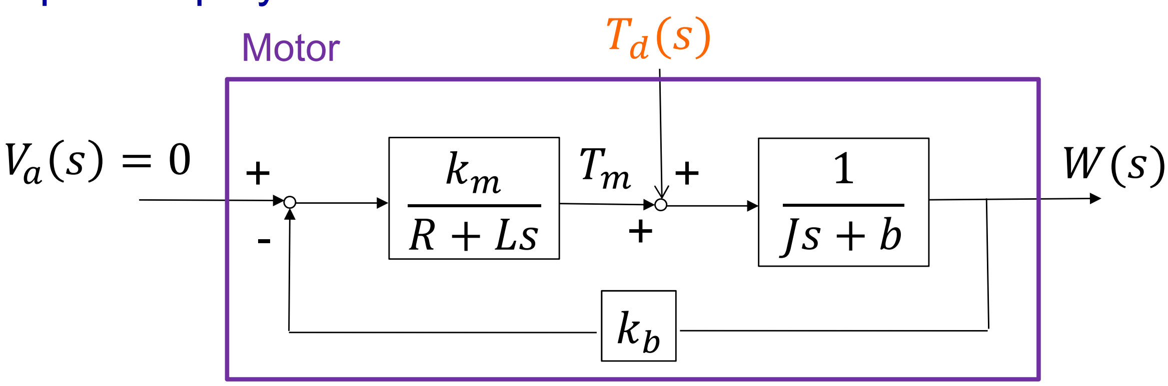
$$G = motor\ plant = \frac{W}{V_a} = \frac{k_m}{(R+Ls)(Js+b)+k_b}$$

2nd-order system

1st-order system if L=0

Disturbance -3

- (1) Open-loop system



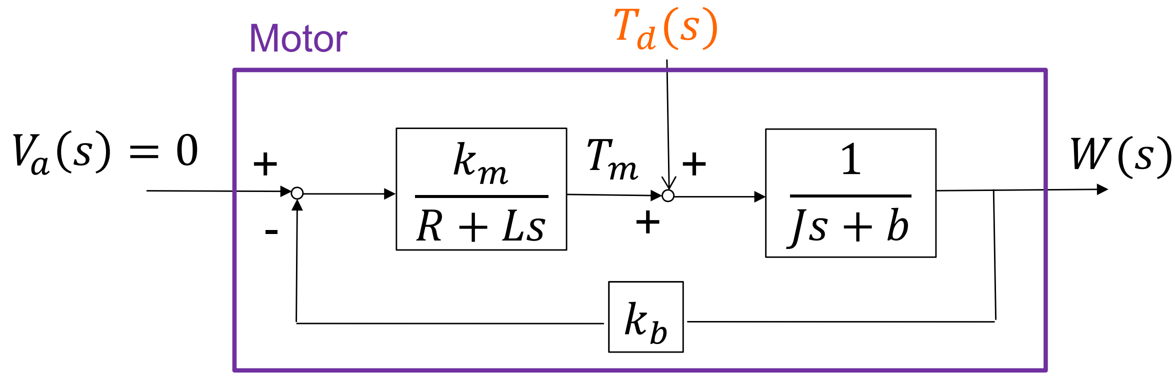
$$E = R - Y = V_a - W = 0 - W = -W$$

$$= -\frac{\frac{1}{Js+b}}{1+\frac{1}{Js+b}k_b\frac{k_m}{R}}T_d = -\frac{1}{Js+(b+\frac{k_bk_m}{R})}T_d$$

(Assume L=0) 1st-order system

Disturbance -4

- (1) Open-loop system



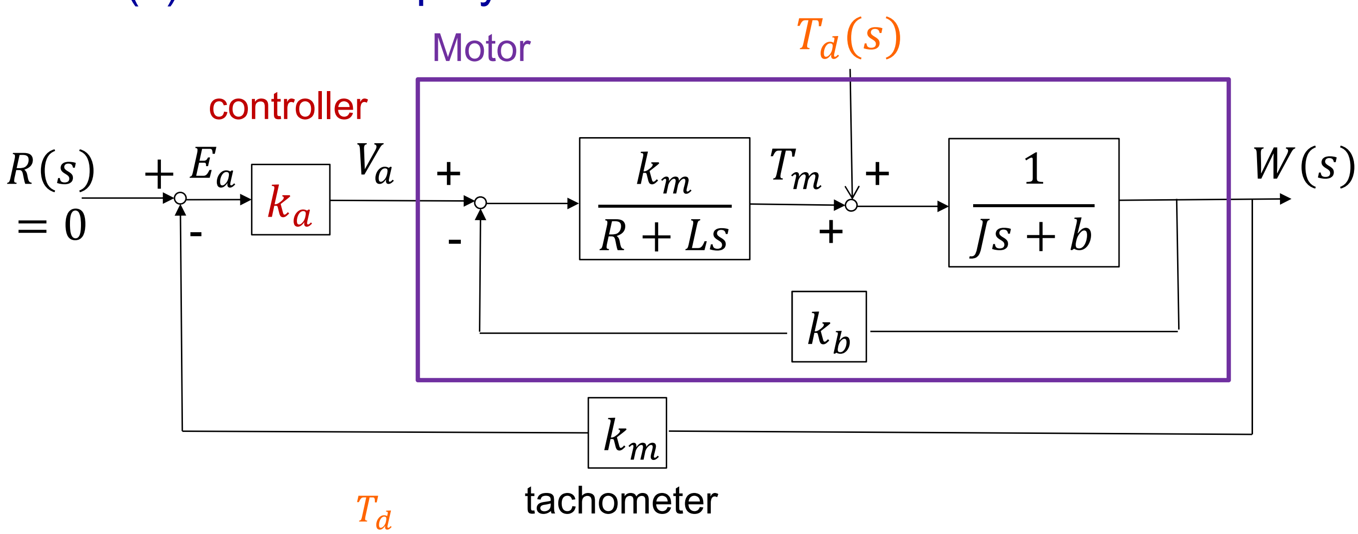
Assume T_d step input $\quad T_d(s) = \frac{D}{s}$

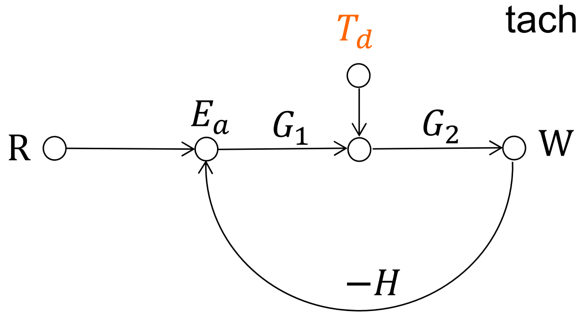
$$\omega_o(\infty) = \lim_{t\to\infty} \omega_o(t) = \lim_{s\to 0} sW_o(s)$$

$$= \lim_{s\to 0} s \frac{1}{Js + (b + k_b k_m/R)} \frac{D}{s} = \frac{R}{bR + k_m k_b} D$$

Disturbance -5

- (2) Closed-loop system





$$G_1 = k_a \frac{k_m}{R} \quad \text{(Assume L=0)}$$

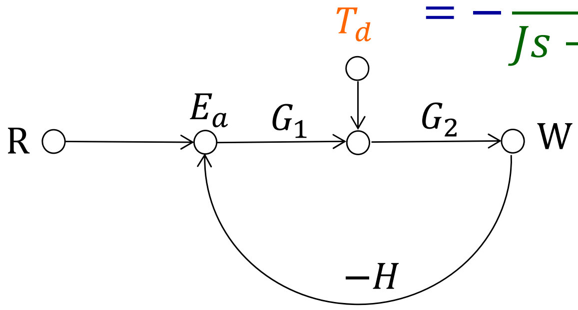
$$G_2 = \frac{1}{Js+b} \qquad H = k_t + \frac{k_b}{k_a}$$

Disturbance -6

$$E = R - Y = 0 - W = -W = -\frac{G_2}{1 + G_2 H G_1} T_d$$

$$= -\frac{1}{Js + (b + (k_t k_a + k_b) k_m / R)} T_d$$

1st-order system



Assume T_d step input $\quad T_d(s) = \frac{D}{s}$

$$\omega_c(\infty) = \lim_{t\to\infty} \omega_c(t) = \lim_{s\to 0} sW_c(s)$$

$$= -\frac{D}{b + (k_t k_a + k_b) k_m / R} \approx \frac{R}{k_m k_t k_a} D \quad \text{(large } k_a\text{)}$$

Disturbance -7

- Open-loop v.s. closed-loop

$$\frac{W_c(\infty)}{W_0(\infty)} = \frac{\frac{R}{k_m k_t k_a} D}{\frac{R}{bR + k_m k_b} D} = \frac{bR + k_m k_b}{k_m k_t k_a} \quad \text{usually} < 0.02$$

Disturbance rejection!

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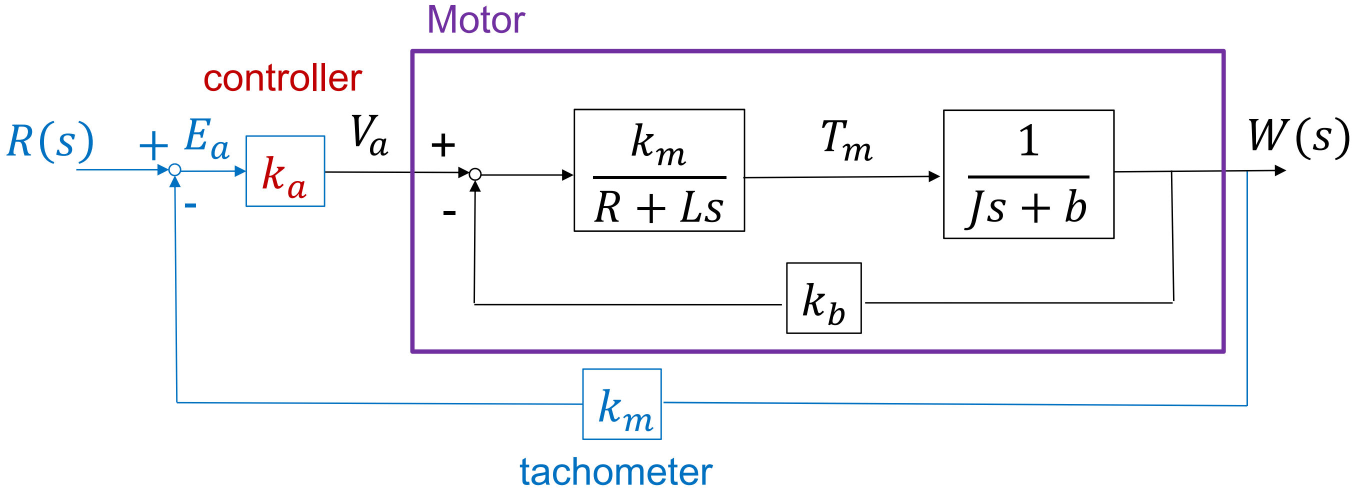
Control of the Transient Response -1

Definition: The response that disappears with time

- EX: motor speed-control



Control of the Transient Response -2

- (1) Open-loop system

$$T_0 = \frac{W_0}{V_a} = \frac{k_m}{R(Js+b)+K_b k_m} = \frac{k_1}{\tau_1 s + 1}$$ (Assume L=0)

where $\tau_1 = \frac{RJ}{Rb+k_m k_b}$ $k_1 = \frac{k_m}{Rb+k_m k_b}$

Assume V_a step input $V_a = \frac{k_2 E}{s}$

$$W_0(s) = \frac{k_1 k_2 E}{s(\tau_1 s+1)} = k_1(k_2 E)[\frac{1}{s} + \frac{-1}{s+\frac{1}{\tau_1}}]$$

$$w_0(t) = k_1 k_2 E(1 - e^{-\frac{t}{\tau_1}})$$

Control of the Transient Response -3

- (2) Closed-loop system

$$T_c = \frac{W_c}{R} = \frac{k_a G}{1+k_a k_t G} = \frac{k_a k_1}{(\tau_1 s+1)+k_a k_t k_1}$$

where $G = \frac{W_0}{V_a} = \mathrm{T}_0$

Assume R step input $R = \frac{k_2 E}{s}$

$$W_c(s) = \frac{k_1 k_2 k_a E}{s[(\tau_1 s+1)+k_1 k_t k_1]}$$

$$w_c(t) = \frac{k_a}{1+k_a k_t k_1}(k_1 k_2 E)(1 - e^{-\frac{t}{p}})$$

where $p = \frac{\tau_1}{1+k_a k_t k_1}$

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Steady-state Error -1

Steady-state response: The response that exists for a long time

- Open-loop v.s. closed-loop

Assume R unit step input $R = \frac{1}{s}$

$$E_0(s) = R - Y = (1 - G)R$$

$$e_0(\infty) = \lim_{t\to\infty} e_0(t) = \lim_{s\to 0} sE_0(s) = \lim_{s\to 0} s(1-G)\underbrace{\frac{1}{s}}_{\text{Unit step input}} = 1 - G(0)$$

$$E_c(s) = R - Y = \frac{1}{1 + G_cG} R$$

Note: $T = \frac{G_cG}{1+G_cG}$ (assume $\mathrm{H(s)=1}$)

$$e_c(\infty) = \lim_{s\to 0} s \frac{1}{1 + G_cG} \underbrace{\frac{1}{s}}_{\text{Unit step input}} = \frac{1}{1 + G_c(0)G(0)}$$

$G(s)\Big|_{s=0}$: $DC\ gain$

Steady-state Error -2

- Ex: A 1st-order system $G = \dfrac{k}{\tau s + 1}$ with $R = \frac{1}{s}$

$$e_0(\infty) = 1 - G(0) = 1 - k \qquad \rightarrow want\ k = 1$$

$$e_c(\infty) = \frac{1}{1 + G_c(0)G(0)} = \frac{1}{1+k} \quad \rightarrow want\ large\ k$$

$$if\ k \rightarrow k + \Delta k$$

$$\Delta e_0(\infty) + e_o(\infty) = 1 - (k + \Delta k) \qquad \Delta e_0(\infty) = -\Delta k = |\Delta k|$$

$$\Delta e_c(\infty) + e_c(\infty) = \frac{1}{1 + (k + \Delta k)} \qquad \Delta e_c(\infty) = \frac{1}{1+k} - \frac{1}{1 + (k+\Delta k)}$$

$$= \frac{1}{(1+k)(1+k+\Delta k)}$$

Less sensitive

Steady-state Error -3

- Ex: A 1st-order system $G = \dfrac{k}{\tau s + 1}$ with $R = \frac{1}{s}$

Assume $\frac{\Delta k}{k} = 0.1\ (10\%)$

if $k = 1$

$$\frac{|e_0|}{|r(t)|} = 0 \qquad \frac{|\Delta e_0|}{|r(t)|} = 10\%$$

$$S_k^G = 1$$

if $k = 100$

$$\frac{|e_c|}{|r(t)|} = \sim 1\% \qquad \frac{|\Delta e_c|}{|r(t)|} = 0.11\%$$

$$S_k^T = S_G^T S_k^G = \frac{1}{(1+G_cG)} 1 = \frac{1}{1+G_cG} = \frac{1}{1+G_c\frac{k}{\tau s+1}}$$

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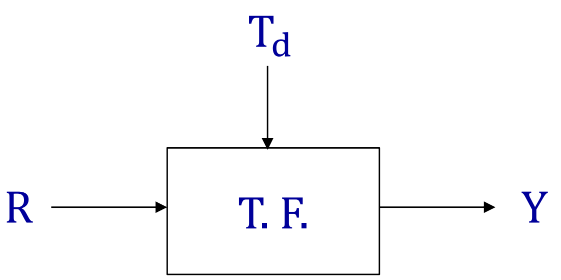
The Cost of Feedback -1

- Summary



y(t) (or e(t))
transient
steady-state

Open-loop vs. closed-loop
sensitivity

The Cost of Feedback -2

- Advantage

1. System sensitivity reduction
2. Transient response improvement
3. Disturbance/noise rejection
4. Steady-state error (and its sensitivity) reduction

The Cost of Feedback -3

- Disadvantage

1. Complexity
2. Loss of gain
3. Measurement noise introduction
4. Possible of instability Chap 6

Open-loop Closed-loop

$$G(s) \quad \rightarrow \quad \frac{G}{1+G}$$

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- Questions?