Chap 3 Manipulator Kinematics

林沛群
國立台灣大學
機械工程學系

Introduction

- Kinematics: The science of motion that treats the subjects without regard to the forces that cause it
  (position/orientation, velocity, acceleration…)
  Forward Kinematics: Chap 3 Chap 5 Chap 6
  Inverse Kinematics: Chap 4

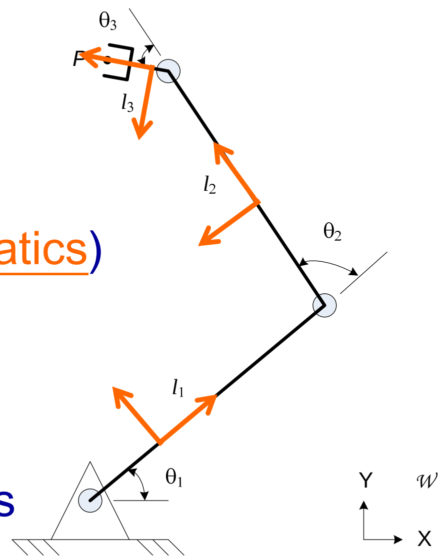
- Dynamics: The relationships between these motions and the forces/torques that cause them
  Chap 6

Manipulator

- Characteristics
  - Complex configuration and mechanism
  - Actuators are defined in local frames
  - Revolute joint or prismatic joint
- What we want to know (Forward kinematics)
  - how θ_i affect P defined in the world frame

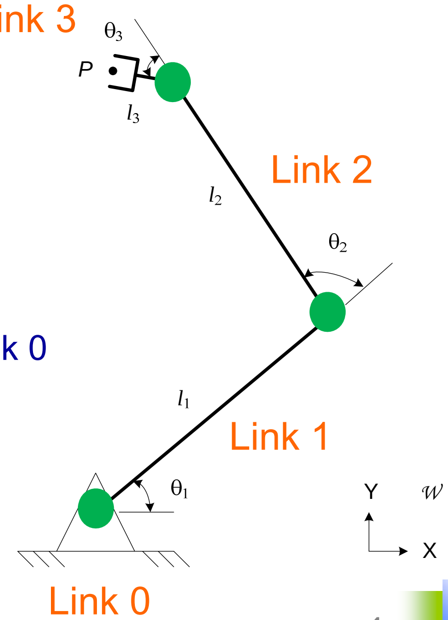
$$^{W}P = f(\theta_1, \theta_2, \ldots, \theta_n)$$

- Approach: Affixing frames to the various parts of the manipulator and describes their relations



Link Description -1

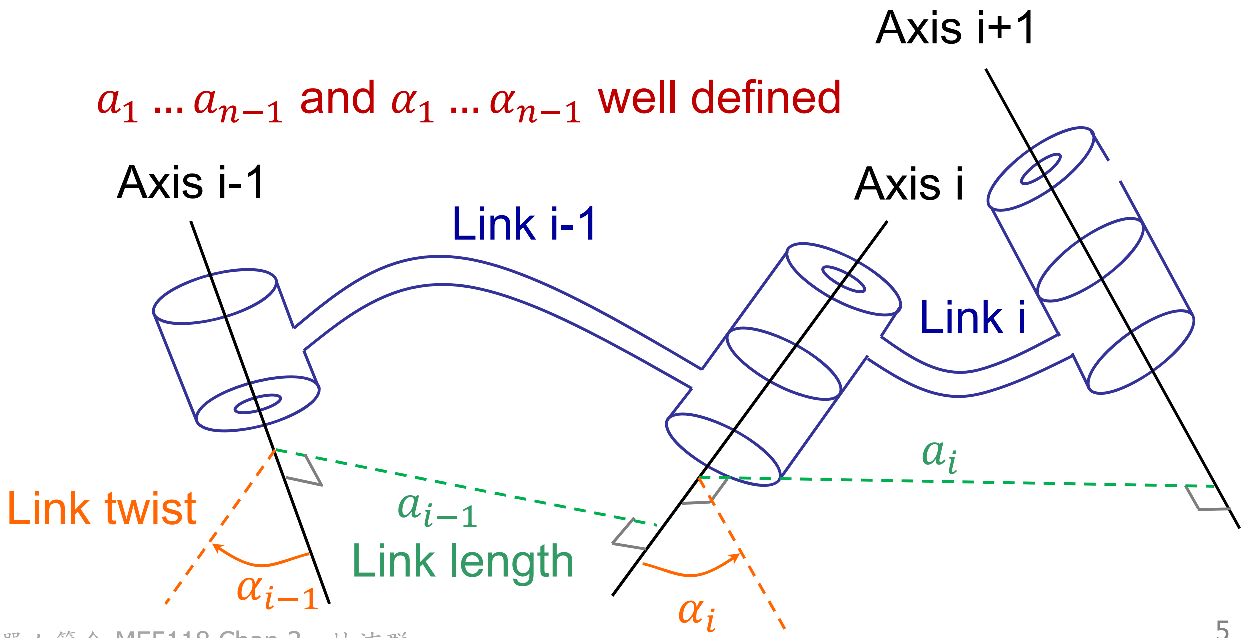
- Joint
  - Each revolute or prismatic joint has 1 DOF
  - Rotate about or move along an "axis"
- Link
  - The rigid body which connects joints
  - Numbering:
    - Link 0: immobile base
    - Link 1: first moving link, connecting to Link 0
    - Link 2: second moving link
    - And so on…



Link Description -2

- For any two axes in 3-space, there exists a well-defined measure of distance between them--- mutually perpendicular to both axes

$a_1 \ldots a_{n-1}$ and $\alpha_1 \ldots \alpha_{n-1}$ well defined



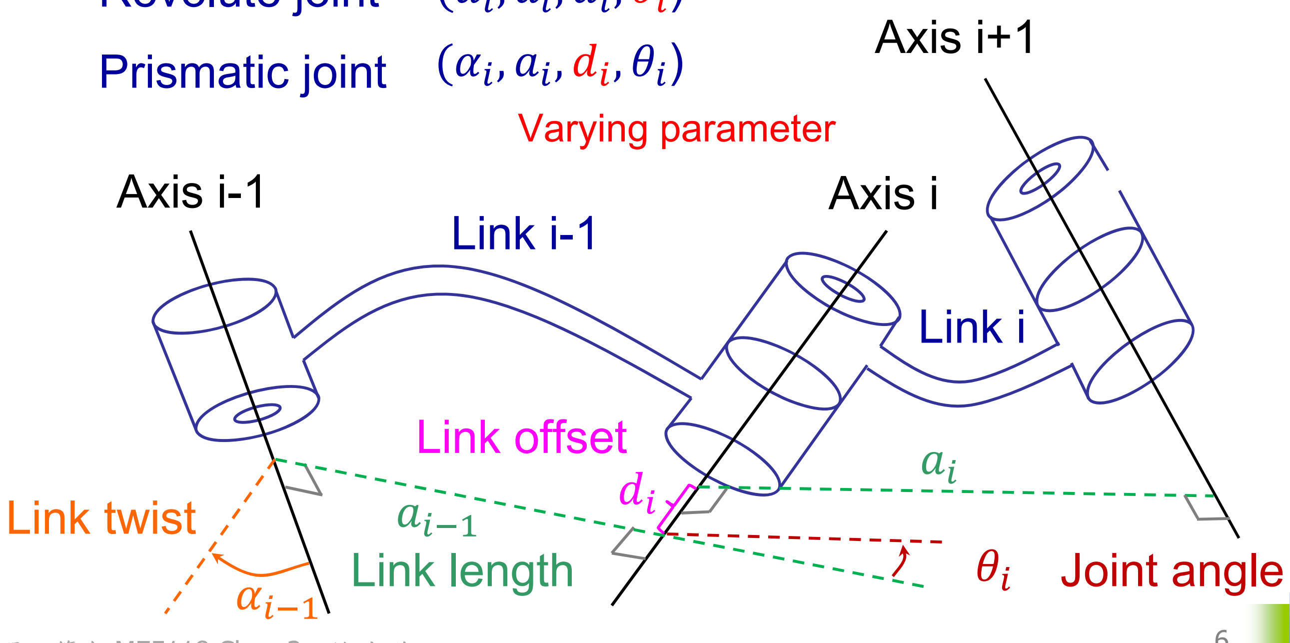
Link Connection Description

- Need two more parameters to define the relation between neighboring links

Revolute joint $(\alpha_i, a_i, d_i, \theta_i)$

Prismatic joint $(\alpha_i, a_i, d_i, \theta_i)$

Varying parameter

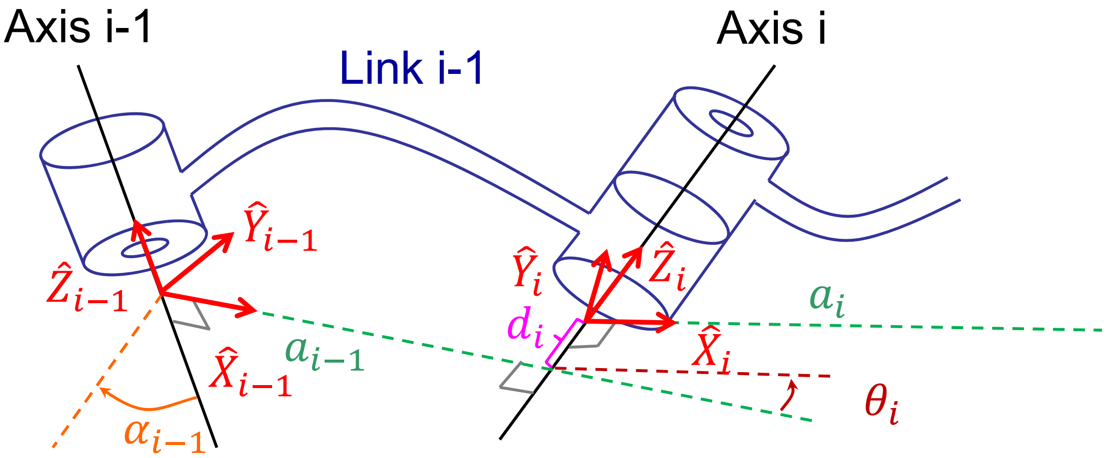


Affixing Frames to Links -1

- $\hat{Z}_i$ Coincident with joint axis

  $\hat{X}_i$ Along a_i (if $a_i \neq 0$)

  Perpendicular to $\hat{Z}_{i-1}$ and $\hat{Z}_i$ (if $a_i = 0$)



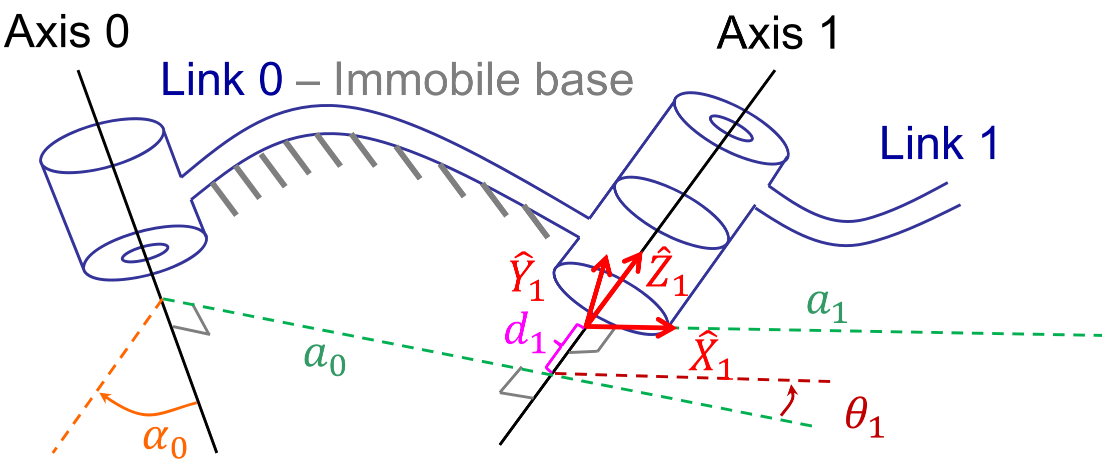
Affixing Frames to Links -2

- First link (0)

  Frame {0} coincides with frame {1} $a_0 = 0$ $\alpha_0 = 0$

  Revolute joint θ_1 arbitrary $d_1 = 0$

  Prismatic joint d_1 arbitrary $\theta_1 = 0$



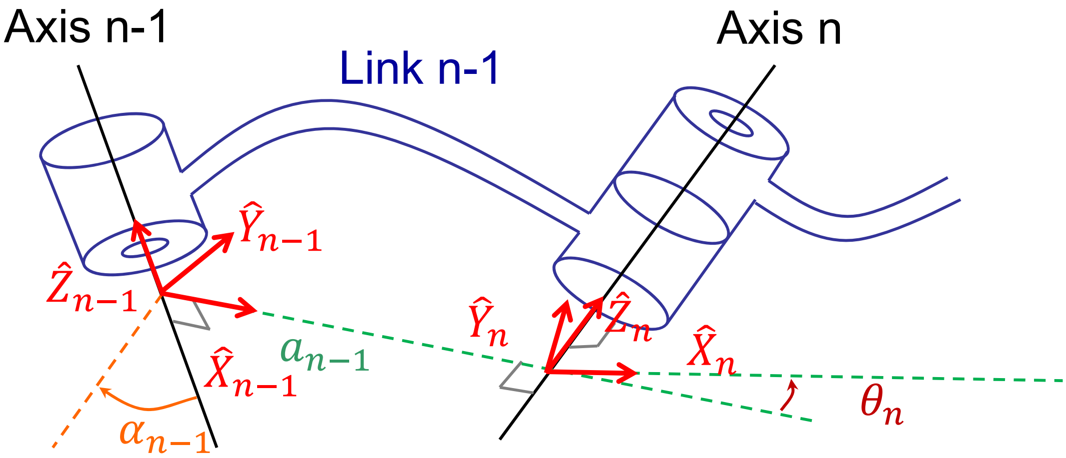
Affixing Frames to Links -2

- Last link (n)

  Extend $\hat{X}_{n-1}$ vector $a_n = 0$ $\alpha_n = 0$

  Revolute joint θ_n variable $d_n = 0$

  Prismatic joint d_n variable $\theta_n = 0$



Summary of DH Notation (Craig)

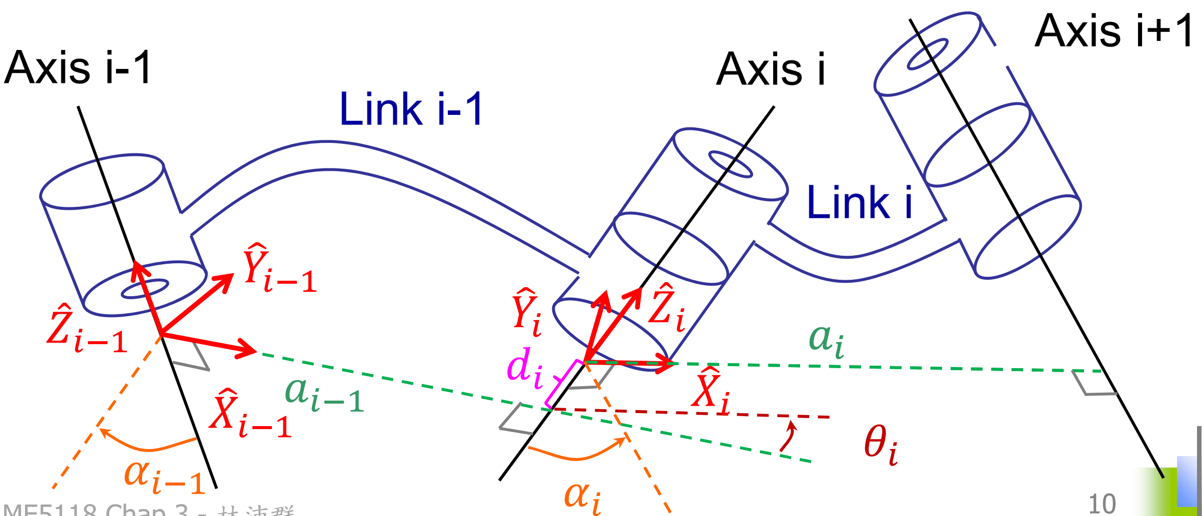
Denavit-Hartenberg

- a_i: the distance from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured along $\hat{X}_i$ ($a_i > 0$)

  α_i: the angle from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured about $\hat{X}_i$

  d_i: the distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$

  θ_i: the angle from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured about $\hat{Z}_i$

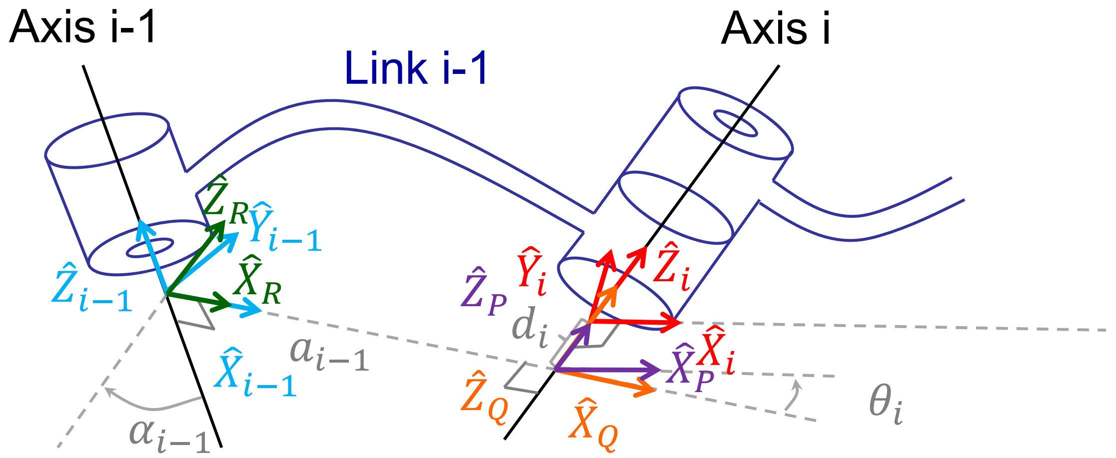


Derivation of Link Transformations -1

- $^{i-1}P = {}^{i-1}_{\ \ i}T\,{}^{i}P$

  $^{i-1}P = {}^{i-1}_{\ \ R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T\,{}^{i}P$

$$
\begin{aligned}
{}^{i-1}_{\ \ i}T &= {}^{i-1}_{\ \ R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T \\
&= T_{\hat{X}_{i-1}}(\alpha_{i-1})\,T_{\hat{X}_R}(a_{i-1})\,T_{\hat{Z}_Q}(\theta_i)\,T_{\hat{Z}_P}(d_i)
\end{aligned}
$$



Derivation of Link Transformations -2

- Thus

$$
{}^{i-1}_{\ \ i}T = T_{\hat{X}_{i-1}}(\alpha_{i-1})\,T_{\hat{X}_R}(a_{i-1})\,T_{\hat{Z}_Q}(\theta_i)\,T_{\hat{Z}_P}(d_i)
$$

$$
= \begin{bmatrix}
c\theta_i & -s\theta_i & 0 & a_{i-1} \\
s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\
s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

- Concatenating link transformations

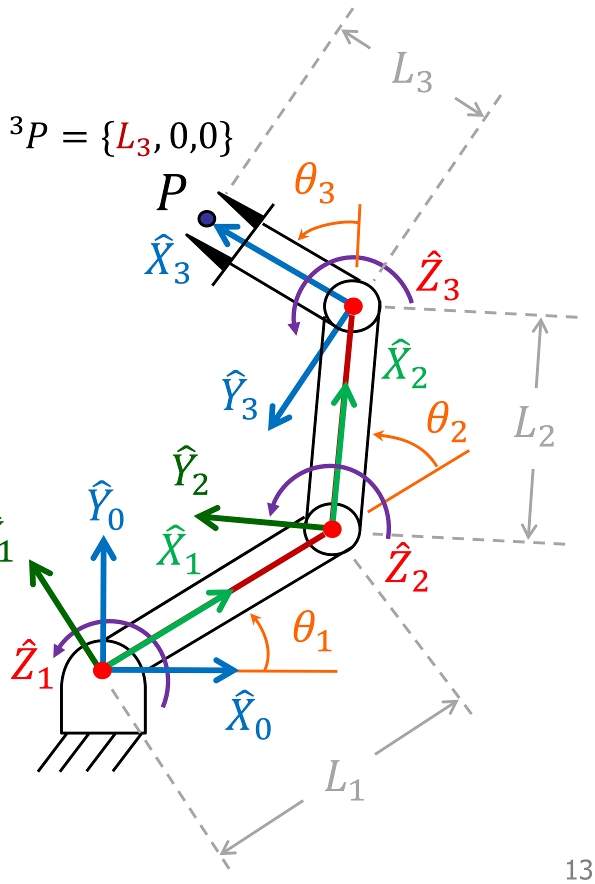
$$
{}^{0}_{n}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T \ldots {}^{n-2}_{n-1}T\,{}^{n-1}_{\ \ n}T
$$

Frame {n} 相對於 Frame {0} 的空間幾何關係具清楚且量化之定義

在Frame {n} 下表達的向量可轉回 Frame {0} 下來表達

Example: A RRR Manipulator

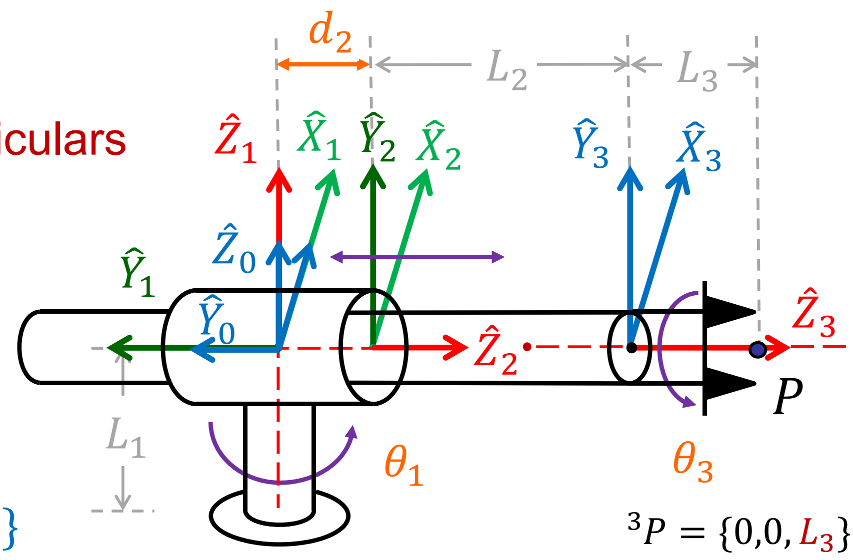
- Joint axes
- Common perpendiculars
- $\hat{Z}_i$
- $\hat{X}_i$
- $\hat{Y}_i$
- Frames $\{0\}$ and $\{n\}$



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

Example: A RPR Manipulator

- Joint axes
- Common perpendiculars
- $\hat{Z}_i$
- $\hat{X}_i$
- $\hat{Y}_i$
- Frames $\{0\}$ and $\{n\}$



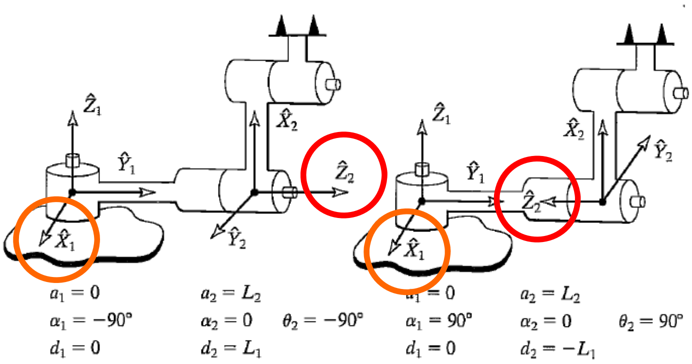
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

Example: RRR Manipulator

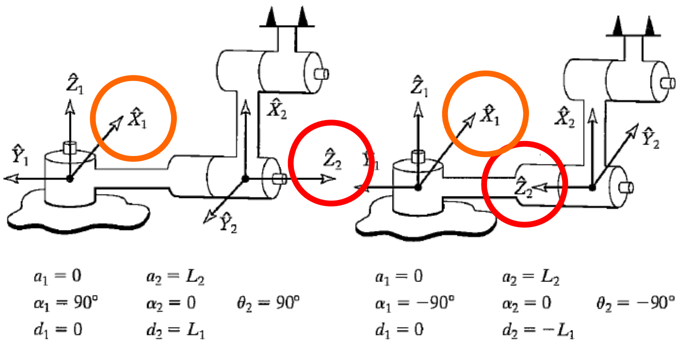
- $a_1 = 0$

  $\hat{Z}_1$ and $\hat{Z}_2$ intersect

  - Two choices for $\hat{Z}_2$

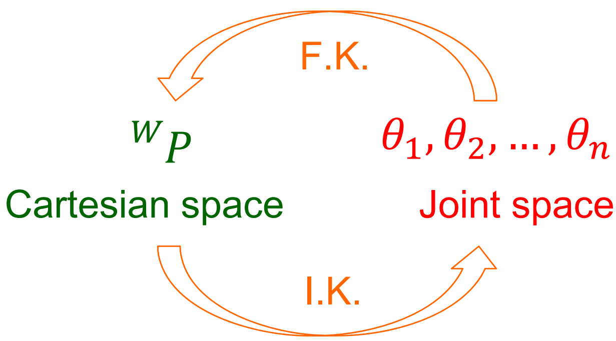


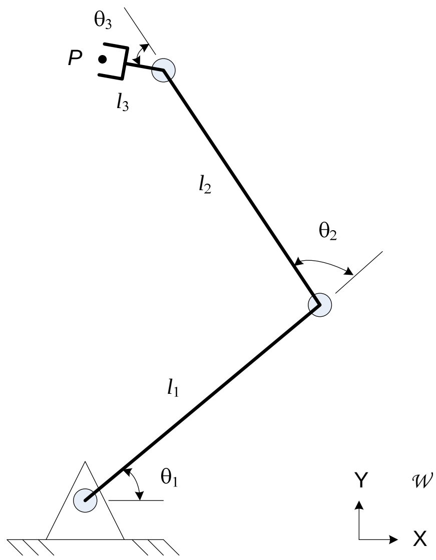
  - Two choices for $\hat{X}_1$



Actuator, Joint, and Cartesian Spaces -1

- Joint space ⇔ Cartesian space





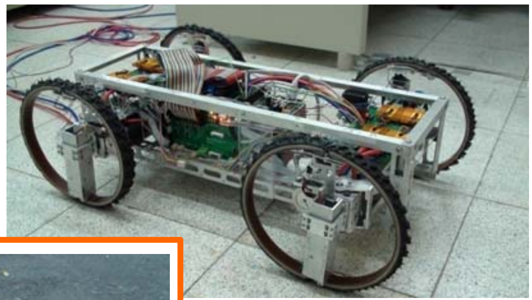
- Actuator space ⇔ joint space
  - Determined by mechanisms which transmits the motion from the actuator to the joint

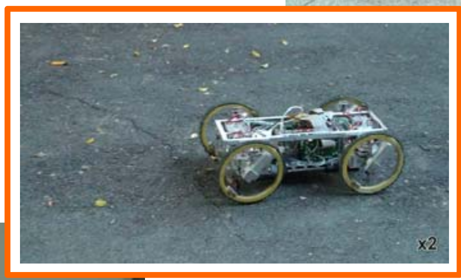
Actuator, Joint, and Cartesian Spaces -2

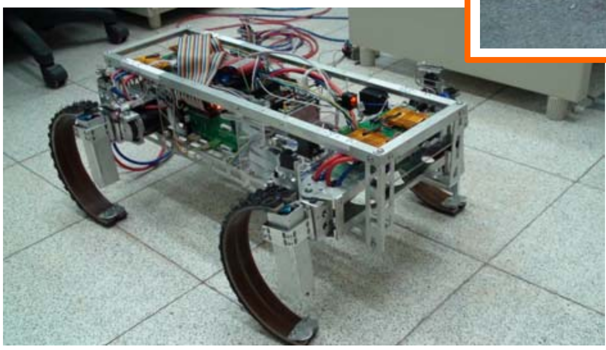
- Example: A leg-wheel transformable robot

Wheel

Fast, smooth, and power-efficient motion on flat ground





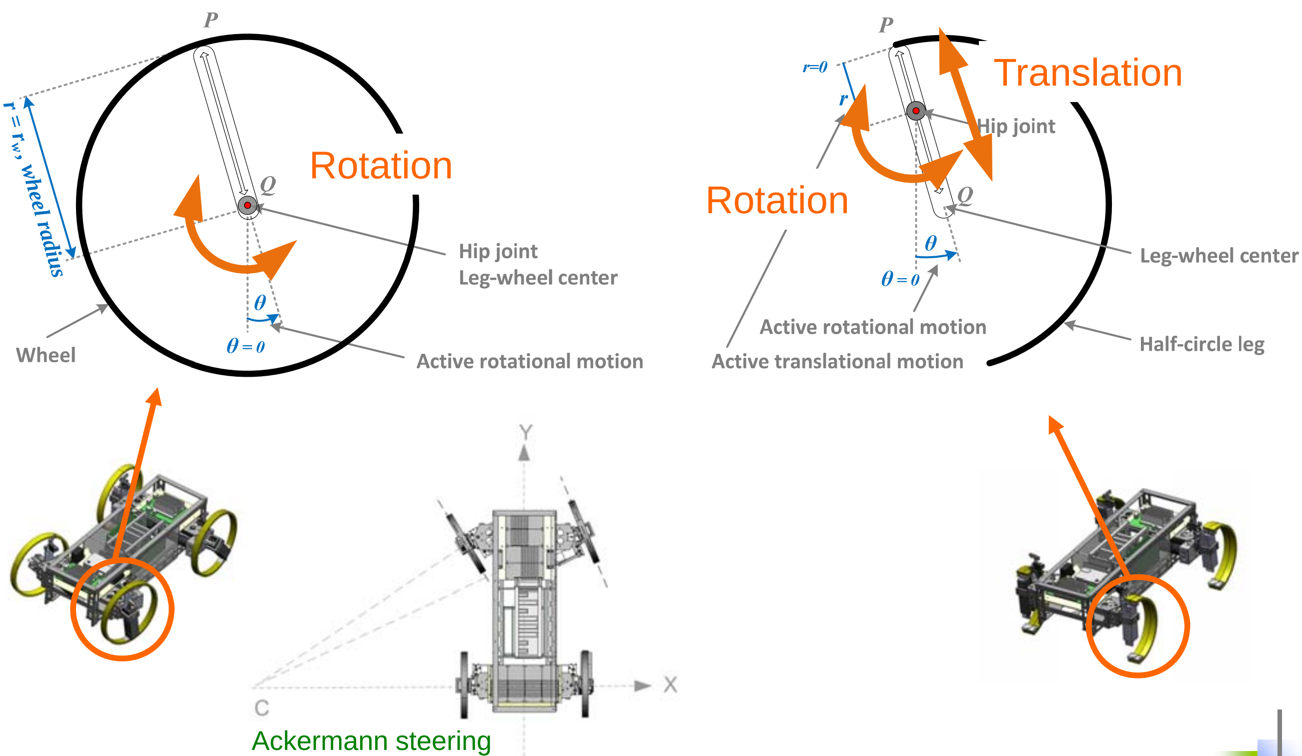


Leg

Rough terrain negotiability

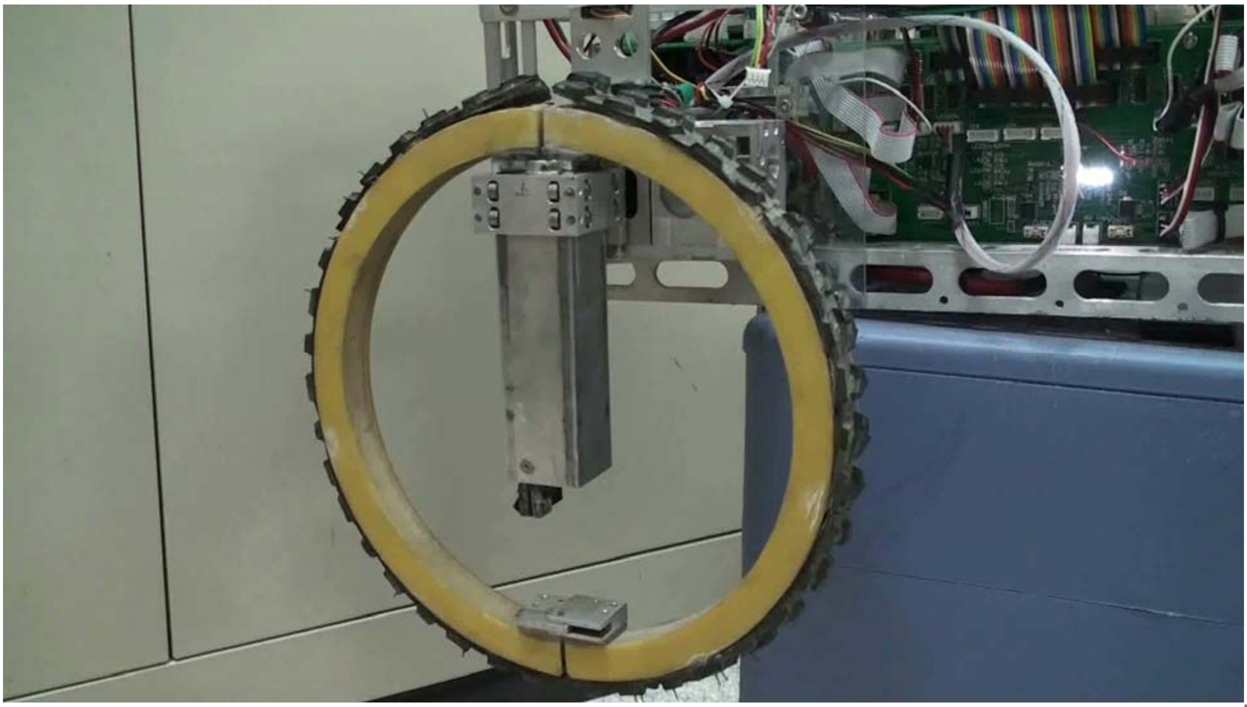
Actuator, Joint, and Cartesian Spaces -3

- Wheeled mode Legged mode



Actuator, Joint, and Cartesian Spaces -4

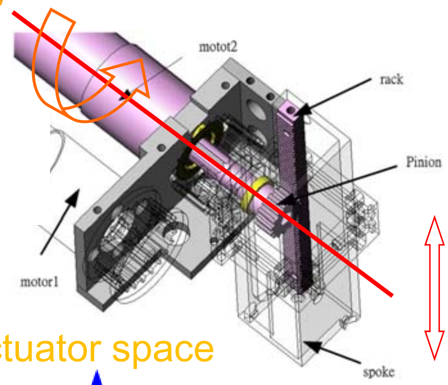
- Leg-wheel motion



Actuator, Joint, and Cartesian Spaces -5

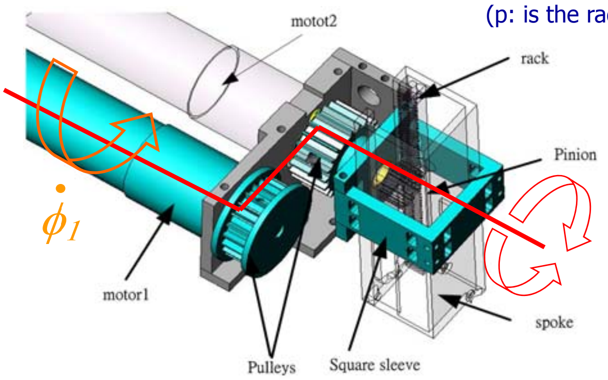
- Kinematic mapping
  - Input: Motor speeds $\dot{\phi}_1$ $\dot{\phi}_2$
  - Output: Leg-wheel motion $\dot{\theta}$ $\dot{r}$

    in polar coordinate

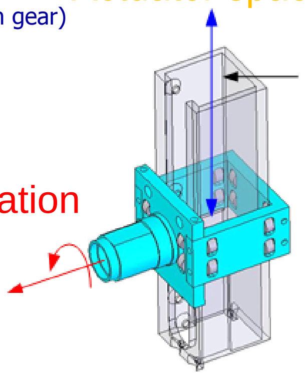


Joint space $\dot{\xi} = \begin{bmatrix} \dot{\theta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -p & p \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix}$ Actuator space

(p: is the radius of pinion gear)



$\dot{\theta}$ Rotation



Translation $\dot{r}$

Summary of DH Notation (Craig) -1

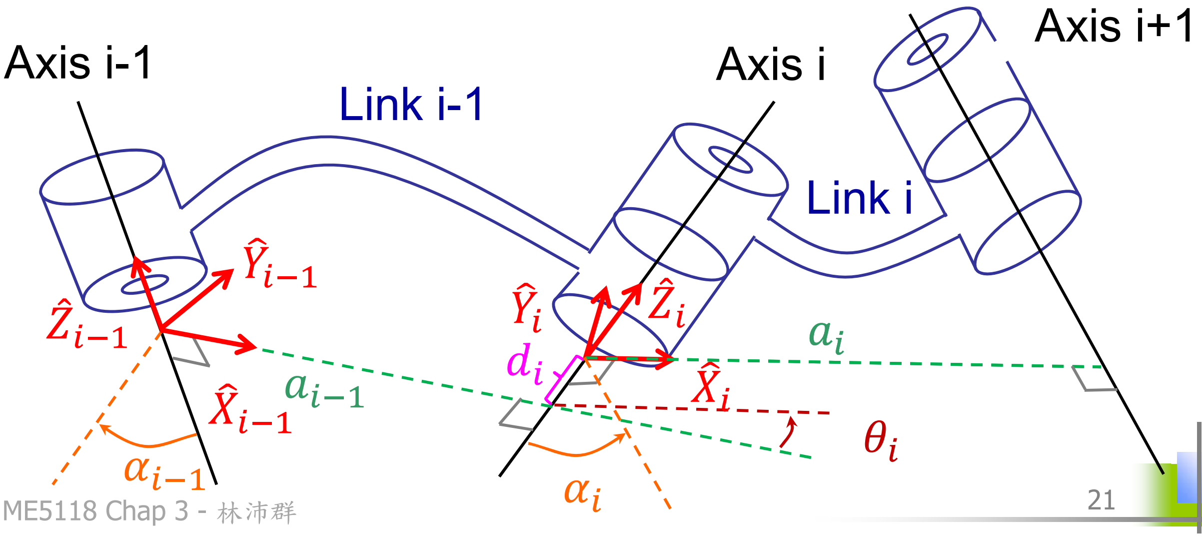
Denavit-Hartenberg

- a_i: the distance from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured along $\hat{X}_i$ ($a_i > 0$)

  α_i: the angle from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured about $\hat{X}_i$

  d_i: the distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$

  θ_i: the angle from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured about $\hat{Z}_i$

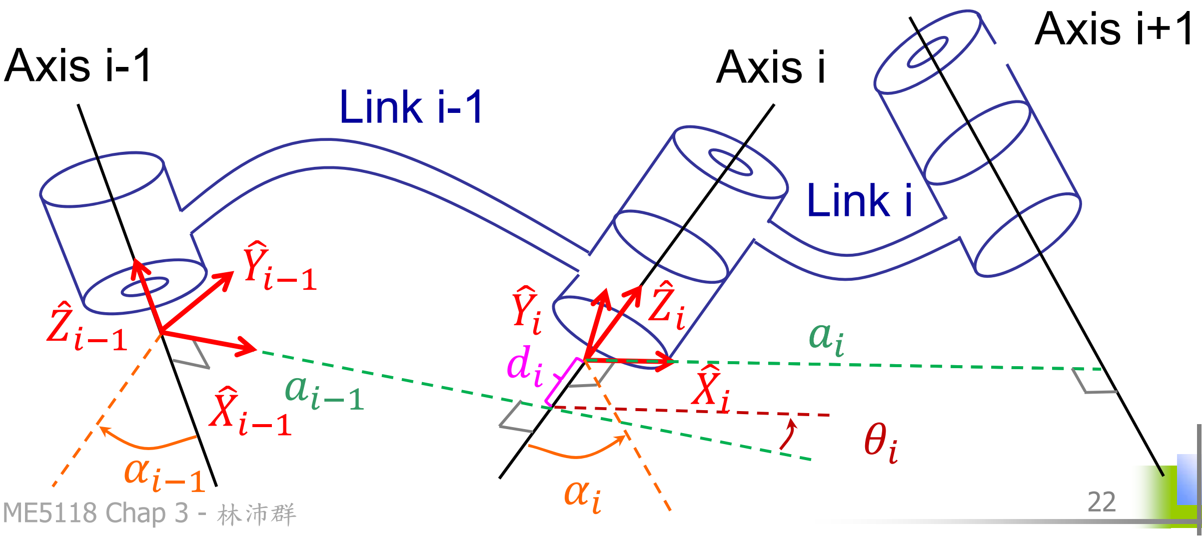


Summary of DH Notation (Craig) -2

- $${}^{i-1}_{i}T = {}^{i-1}_{R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T$$

$$= T_{\hat{X}_{i-1}}(\alpha_{i-1})T_{\hat{X}_R}(a_{i-1})T_{\hat{Z}_Q}(\theta_i)T_{\hat{Z}_P}(d_i)$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Summary of DH Notation (Standard) -1

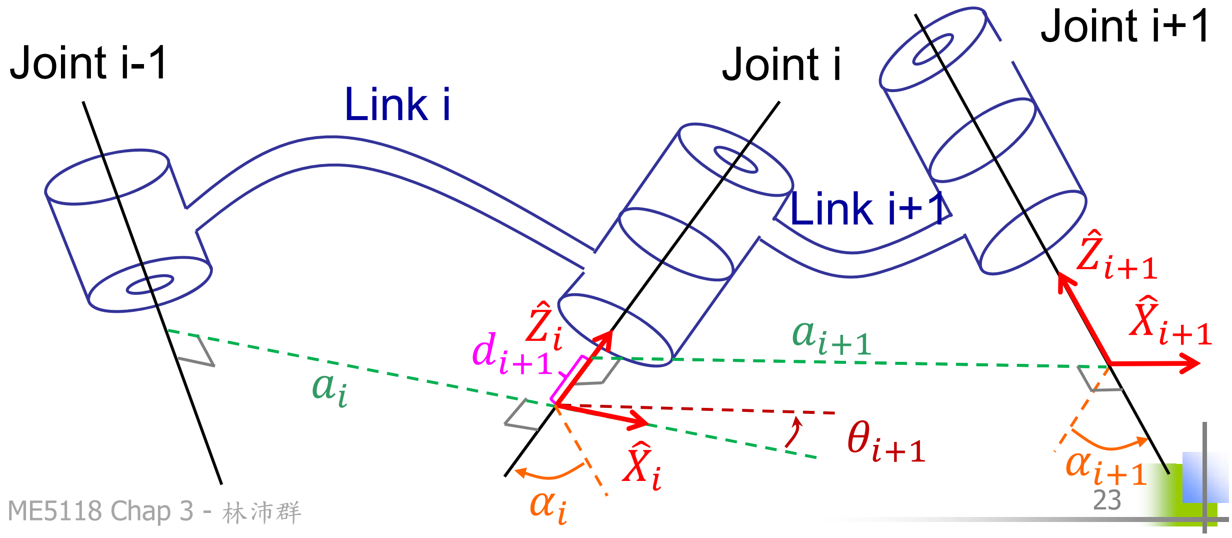
Denavit-Hartenberg

- θ_i: the distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured about $\hat{Z}_{i-1}$

  d_i: the distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_{i-1}$

  a_i: the distance from $\hat{Z}_{i-1}$ to $\hat{Z}_i$ measured along $\hat{X}_i$

  α_i: the distance from $\hat{Z}_{i-1}$ to $\hat{Z}_i$ measured about $\hat{X}_i$

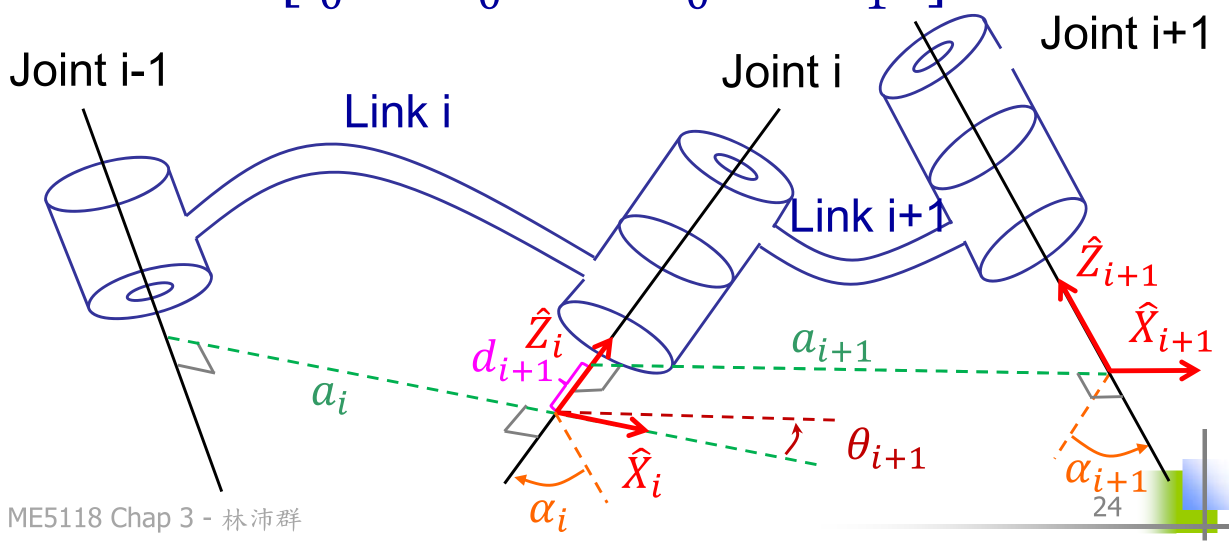


Summary of DH Notation (Standard) -2

- $$ {}^{i-1}_{i}T = {}^{i-1}_{R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T $$

  $$ = T_{\hat{Z}_{i-1}}(\theta_i)T_{\hat{Z}_R}(d_i)T_{\hat{X}_Q}(a_i)T_{\hat{X}_P}(\alpha_i) $$

  $$ = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

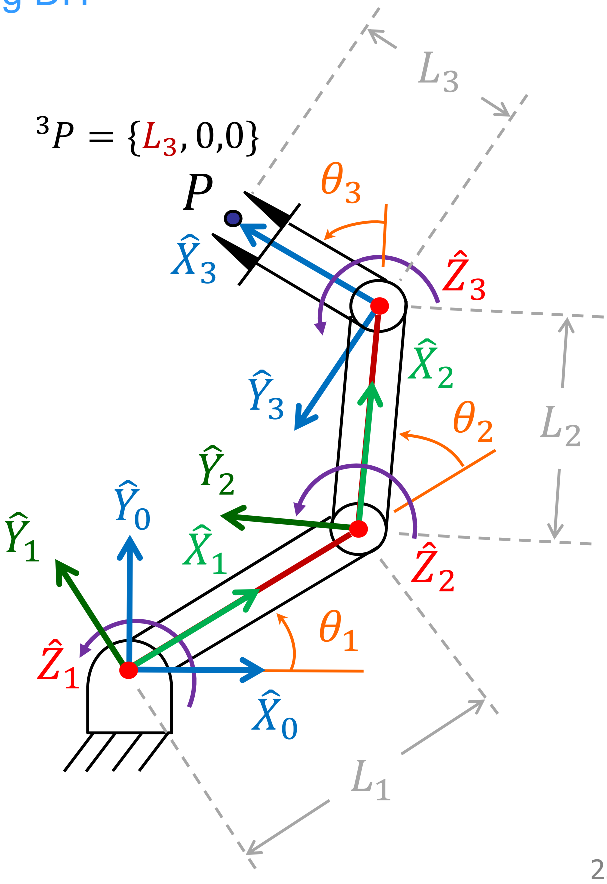


Revisit Example: A RRR Manipulator -1

Craig DH

- Joint axes
- Common perpendiculars
- $\hat{Z}_i$
- $\hat{X}_i$
- $\hat{Y}_i$
- Frames $\{0\}$ and $\{n\}$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3



Revisit Example: A RRR Manipulator -2

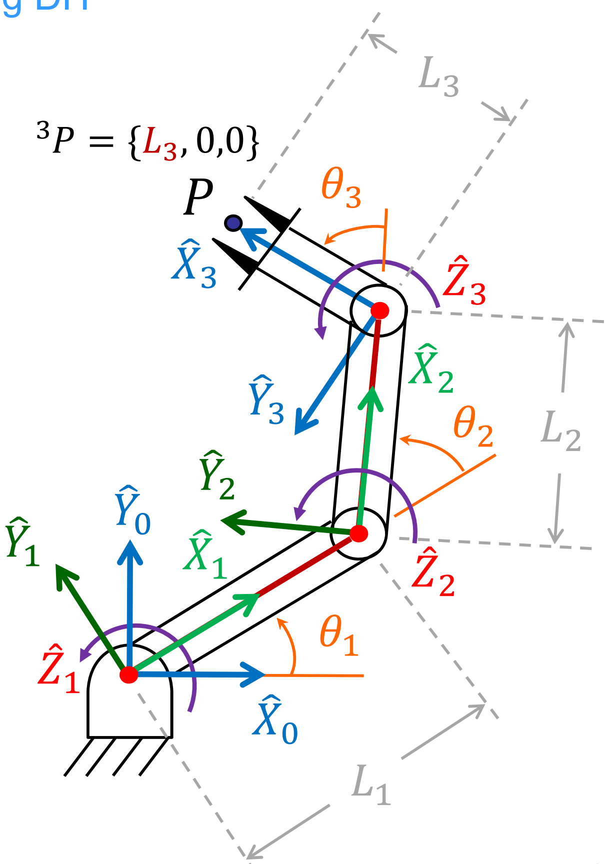
Craig DH

- Transformation matrices

$${}^0_1T \quad \begin{pmatrix} \text{Cos[t1]} & -\text{Sin[t1]} & 0 & 0 \\ \text{Sin[t1]} & \text{Cos[t1]} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^1_2T \quad \begin{pmatrix} \text{Cos[t2]} & -\text{Sin[t2]} & 0 & \text{L1} \\ \text{Sin[t2]} & \text{Cos[t2]} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^2_3T \quad \begin{pmatrix} \text{Cos[t3]} & -\text{Sin[t3]} & 0 & \text{L2} \\ \text{Sin[t3]} & \text{Cos[t3]} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

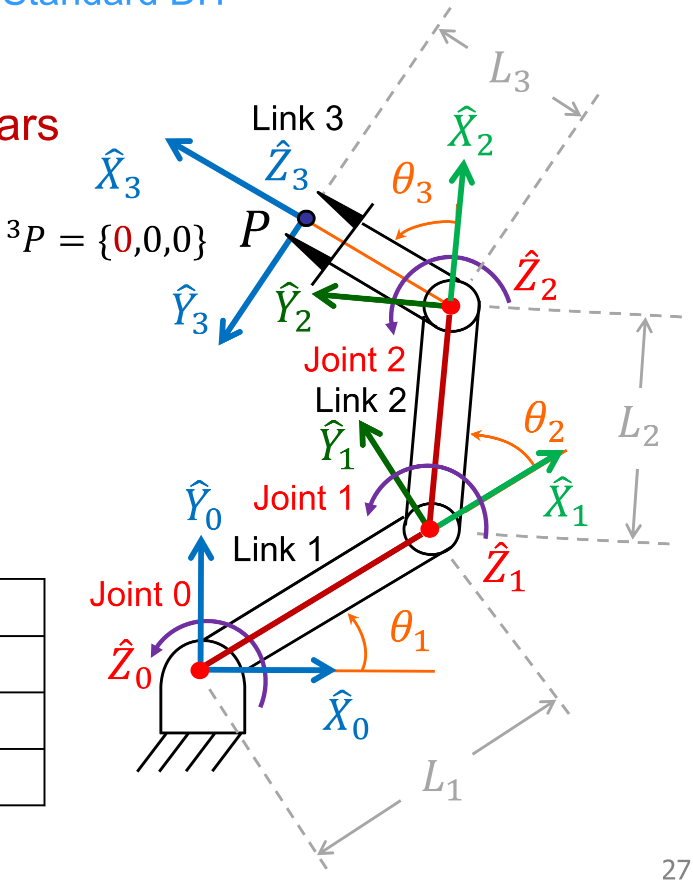


Revisit Example: A RRR Manipulator -3

Standard DH

- Joint axes
- Common perpendiculars
- $\hat{Z}_i$
- $\hat{X}_i$
- $\hat{Y}_i$
- Frames $\{0\}$ and $\{n\}$

i	α_i	a_i	d_i	θ_i
1	0	L_1	0	θ_1
2	0	L_2	0	θ_2
3	0	L_3	0	θ_3



Revisit Example: A RRR Manipulator -4

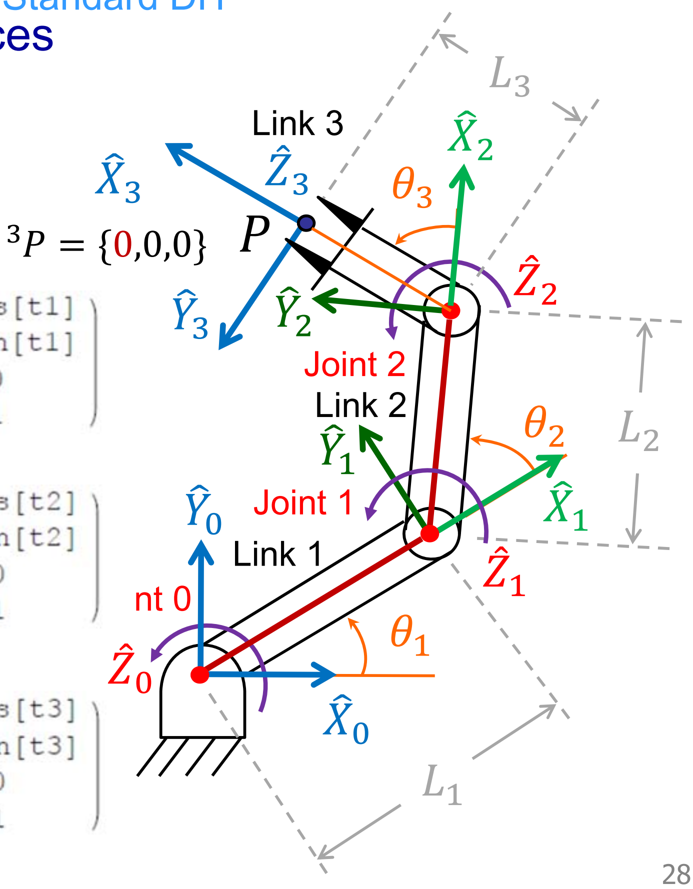
Standard DH

- Transformation matrices

$$ {}^0_1T \quad \begin{pmatrix} \text{Cos[t1]} & -\text{Sin[t1]} & 0 & \text{L1 Cos[t1]} \\ \text{Sin[t1]} & \text{Cos[t1]} & 0 & \text{L1 Sin[t1]} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^1_2T \quad \begin{pmatrix} \text{Cos[t2]} & -\text{Sin[t2]} & 0 & \text{L2 Cos[t2]} \\ \text{Sin[t2]} & \text{Cos[t2]} & 0 & \text{L2 Sin[t2]} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^2_3T \quad \begin{pmatrix} \text{Cos[t3]} & -\text{Sin[t3]} & 0 & \text{L3 Cos[t3]} \\ \text{Sin[t3]} & \text{Cos[t3]} & 0 & \text{L3 Sin[t3]} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$



Revisit Example: A RRR Manipulator -5

- Craig

$$ {}^0_3T = \begin{pmatrix} \mathrm{Cos}[t1+t2+t3] & -\mathrm{Sin}[t1+t2+t3] & 0 & \mathrm{L1\,Cos}[t1] + \mathrm{L2\,Cos}[t1+t2] \\ \mathrm{Sin}[t1+t2+t3] & \mathrm{Cos}[t1+t2+t3] & 0 & \mathrm{L1\,Sin}[t1] + \mathrm{L2\,Sin}[t1+t2] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^0_3T . T_{\hat{X}_3}([L_3, 0,0]\ ) $$

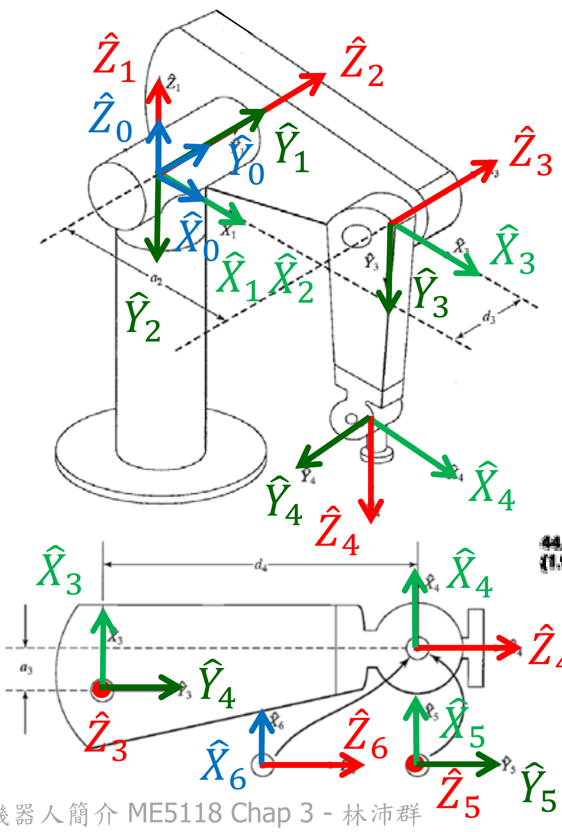
$$ \begin{pmatrix} \mathrm{Cos}[t1+t2+t3] & -\mathrm{Sin}[t1+t2+t3] & 0 & \mathrm{L1\,Cos}[t1] + \mathrm{L2\,Cos}[t1+t2] + \mathrm{L3\,Cos}[t1+t2+t3] \\ \mathrm{Sin}[t1+t2+t3] & \mathrm{Cos}[t1+t2+t3] & 0 & \mathrm{L1\,Sin}[t1] + \mathrm{L2\,Sin}[t1+t2] + \mathrm{L3\,Sin}[t1+t2+t3] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

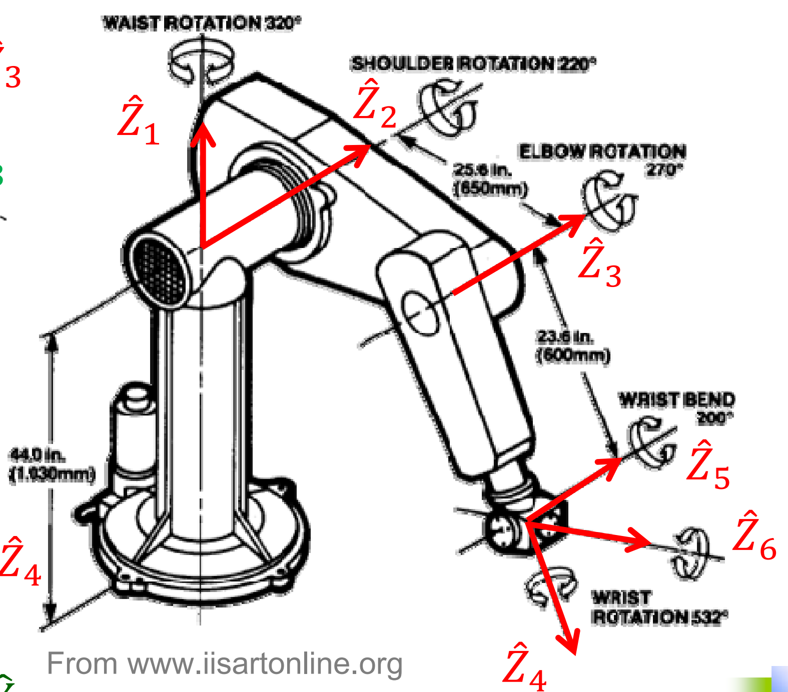
- Standard

$$ {}^0_3T = \begin{pmatrix} \mathrm{Cos}[t1+t2+t3] & -\mathrm{Sin}[t1+t2+t3] & 0 & \mathrm{L1\,Cos}[t1] + \mathrm{L2\,Cos}[t1+t2] + \mathrm{L3\,Cos}[t1+t2+t3] \\ \mathrm{Sin}[t1+t2+t3] & \mathrm{Cos}[t1+t2+t3] & 0 & \mathrm{L1\,Sin}[t1] + \mathrm{L2\,Sin}[t1+t2] + \mathrm{L3\,Sin}[t1+t2+t3] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Example: PUMA 560 -1

- Frames (Craig)

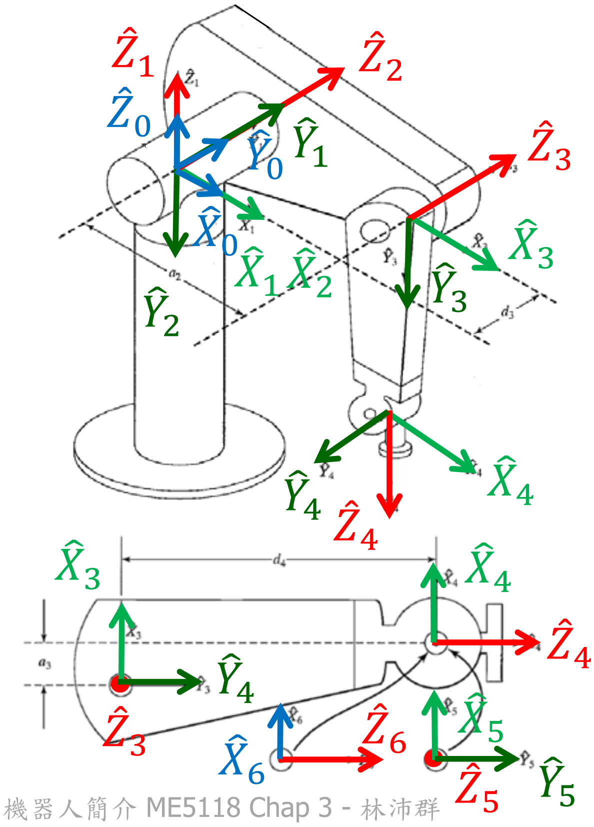




From www.iisartonline.org

Example: PUMA 560 -2

- DH parameters (Craig)



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0°	0	0	θ_1
2	-90°	0	0	θ_2
3	0°	a_2	d_3	θ_3
4	-90°	a_3	d_4	θ_4
5	90°	0	0	θ_5
6	-90°	0	0	θ_6

Example: PUMA 560 -3

- Transformation matrices

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}_{5}T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{5}_{6}T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Example: PUMA 560 -4

- Combining transformation matrices -1

$$ {}^{4}_{6}T = {}^{4}_{5}T\,{}^{5}_{6}T = \begin{bmatrix} c_5c_6 & -c_5s_6 & -s_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{6}T = {}^{3}_{4}T\,{}^{4}_{6}T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & -c_4s_5 & a_3 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{3}T = {}^{1}_{2}T\,{}^{2}_{3}T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_2c_2 \\ 0 & 0 & 1 & d_3 \\ -s_{23} & -c_{23} & 0 & -a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

- Combining transformation matrices -2

$$ {}^{1}_{6}T = {}^{1}_{3}T\,{}^{3}_{6}T = \begin{bmatrix} {}^{1}r_{11} & {}^{1}r_{12} & {}^{1}r_{13} & {}^{1}p_x \\ {}^{1}r_{21} & {}^{1}r_{22} & {}^{1}r_{23} & {}^{1}p_y \\ {}^{1}r_{31} & {}^{1}r_{32} & {}^{1}r_{33} & {}^{1}p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}r_{11} = c_{23}[c_4c_5c_6 - s_4s_6] - s_{23}s_5s_6 $$
$$ {}^{1}r_{21} = -s_4c_5c_6 - c_4s_6 $$
$$ {}^{1}r_{31} = -s_{23}[c_4c_5c_6 - s_4s_6] - c_{23}s_5c_6 $$
$$ {}^{1}r_{12} = -c_{23}[c_4c_5s_6 + s_4c_6] + s_{23}s_5s_6 $$
$$ {}^{1}r_{22} = s_4c_5s_6 - c_4c_6 $$
$$ {}^{1}r_{32} = s_{23}[c_4c_5s_6 + s_4c_6] + c_{23}s_5s_6 $$
$$ {}^{1}r_{13} = -c_{23}c_4s_5 - s_{23}c_5 $$
$$ {}^{1}r_{23} = s_4s_5 $$
$$ {}^{1}r_{33} = s_{23}c_4s_5 - c_{23}c_5 $$
$$ {}^{1}p_x = a_2c_2 + a_3c_{23} - d_4s_{23} $$
$$ {}^{1}p_y = d_3 $$
$$ {}^{1}p_z = -a_3s_{23} - a_2s_2 - d_4c_{23} $$

- Combining transformation matrices -3

$$ {}^0_6T = {}^0_1T\,{}^1_6T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6)$

$r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6] - c_1(s_4c_5c_6 + c_4s_6)$

$r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6)\text{-}c_{23}s_5c_6$

$r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6)$

$r_{22} = s_1[s_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6)$

$r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6$

$r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5)\text{-}\ s_1s_4s_5$

$r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5$

$r_{33} = s_{23}c_4s_5 - c_{23}c_5$

$p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1$

$p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1$

$p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}$

終

- Questions?